Introduction to Linear Algebra for Software Development

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I got my degree in Mathematics at the Universitat Autònoma de Barcelona (UAB) in 2009 and did my eHealth Masters at the McMaster University (Canada) in 2011. While I did my graduate training I was also a teaching assistant in the Department of Computer Sciences at the McMaster University. In 2012, I developed my professional career as a Software Developer working for several companies in the digital sector in the Barcelona area, including Mascoteros (a marketplace for the pet industry) and YaEncontre (a real estate website). In these companies I specialized in web development with Symfony, a popular PHP framework based on MVC architecture. The rapidly changing digital industry was a catalyst which led me to look to new technologies such as Big Data, Computer Vision and Computational Modeling. As a result of my expertise and motivation for research I am currently working as a Data Manager at ICTA (Institut de Ciència i Tecnologia Ambiental, UAB).

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Introduction

This course has two main objectives. First, to provide the fundamental concepts on linear algebra; and second, to illustrate the diversity of applications of linear algebra into different fields and in particular in Information Technology and Software Development.

Mathematical modelling has become a great tool for scientists in all fields to try to understand nature and society. Its contribution has been very important in physics, biology, economy, social sciences and in the development of new technologies. Throughout this course a series of mathematical concepts will be presented. An additional objective of this course is to provide examples of linear algebra as applied in a variety of fields. Through these examples the student will see how similar mathematical models can be used in various different scenarios. In mathematical modelling, the interpretation of results into the specific field is a key step that when applied to organizations can help to burst their value.

The fast growth of digital economy requires more Software Development than ever. E-commerce, e-learning, Digital Marketing and Social Media are some of the new sectors in which software development has become a key element. Platforms in all these sectors benefit from having in-house software developers because this allows them to quickly adapt to continuous changing environments. Nevertheless, in today's world of digital business software developers can provide more than fast software development. With knowledge in mathematical modelling and algorithms, software developers can help businesses to optimize their processes or better understand their customers' needs to provide better customer service.

The content of this course is divided into 5 challenges. The first three challenges focus on mathematical concepts and the last two challenges are applications of the given mathematical concepts.

1. Why is linear algebra important in Computer science and what are its basic elements?
2. How to solve common problems in Computer science with systems of linear equations
3. Eigenvalues, eigenvectors and how Netflix uses them
4. How to face the curse of dimensionality with principal component analysis and singular value decomposition
5. How to model dynamic systems with Markov chains as used by the Google PageRank algorithm

In Challenge 1 we will present basic concepts in linear algebra such as Vector Spaces, Matrices, Determinants and the Inverse Matrix. We will also look at the geometrical interpretation of these mathematical elements.

In Challenge 2 we will see how the elements learned in Challenge 1 will be used to solve systems of linear equations. In addition, we will show a geometrical interpretation of these systems.

In Challenge 3 we will introduce the concept of Linear Map together with two fundamental tools: eigenvalues and eigenvectors. We will also look at diagonalization. These concepts, which are the base of many technologies, will be used throughout the course.

In Challenge 4 we will study two techniques based on Linear Algebra and then apply them to real cases. First, Principal Component Analysis (PCA), which is a very useful analysis when tackling the study of any dataset. Second, Single Value Decomposition (SVD), which is a technique with several applications in signal processing, statistics and image manipulation. To present the two procedures, we will look at detailed examples using mathematical software. Therefore, the student will be able to apply these techniques to everyday problems.

Finally, in Challenge 5 we will focus on Markov Models, which at the very start were only used in probability and statistics to study the Random Walk problem or gambling scenarios. In time, however, Markov Chains turned out to be useful applications in a variety of other fields, from the foundation of Google PageRank algorithms and queueing theory to population dynamics or market analyses.

The objective of this course is to present mathematical concepts that have a practical application for everyday problems that software developers may have to face in their future careers. We will present examples from other disciplines since an additional aim is to make students aware that skilled software developers are creative and capable of transferring ideas from one field to another.

Objectives

The challenges briefly presented in this introduction to the course are part of the teaching resources of this course. The main objectives of each of these challenges are:

1. To understand and be able to use basic concepts of Linear Algebra such as: Vector Spaces, Matrices, Determinants, Scalar or Dot Product as well as the geometric interpretation of these elements.
2. To understand the importance of solving systems of linear equations when facing problems in computer sciences or in closely related fields such as data sciences.
3. To master the calculation and understanding of the meaning of eigenvalues and eigenvectors since they are key in many applications of linear algebra.
4. To understand the Principal Component Analysis (PCA) technique and be able to use it with real data.
5. To use the Single Value Decomposition (SVD) technique to solve everyday problems such as image compression.
6. To be able to use matrices to model different problems by means of Markov Chains.
7. To become familiar with the R programming language to solve everyday mathematical problems.