Interactive Calculus in a Virtual Learning Environment*

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Abstract

The teaching of higher level mathematics for technical students in a virtual learning environment poses some difficulties, but also opportunities, now specific to that virtuality. On the other hand, resources and ways to do now manly available in VLEs might soon extend to all kinds of environments.

In this short presentation we will discuss an experience carried at Universitat Oberta de Catalunya (UOC) involving (an on line university), first, the translation of LaTeX written existent materials to a web based format (specifically, a combination of XHTML and MathML), and then the integration of a symbolic calculator software (WIRIS) running as a Java applet embedded in the materials, intending to achieve an evolution from memorising concepts and repetitive algorithms to understanding and experiment concepts and the use of those algorithms.

1. Introduction

Fast developments in information and communications technologies and changes in the behaviour of learners demand educational institutions to continuously evaluate their pedagogical approaches to the learning and teaching process, both in face-to-face and virtual classrooms. Especially higher education institutions are challenged by great changes caused by multiple external factors such as the increasing difference among relative costs and benefits of the physical versus the virtual environment or the impressive shifts in the characteristics of students and the way they affect their curriculum (McGovern et al., 2005).

On the other hand, during the last two decades we have been able to witness the appearance of a new kind of students, thank to their continuous exposure to digital technologies. These students think and process information differently from the ones whose interaction with these tools not "innate". These new learners inhabit a social, cultural and technological environment where their learning is constructed as a much more social process. A social constructivist view of learning considers that students learn best when they construct their interpretations on a subject and communicate their recently acquired knowledge to others (Gay et al., 2001). Within this context the next generation technologies known under the "web 2.0" name are considered as an opportunity for innovation in teaching and learning (Alexander, 2006).

The addition of both those pedagogical and technological new contexts and advances is known as e-learning 2.0 or education 2.0.

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1.1 Technological context

The talk about 'web 2.0' began in 2004, when the O'Reilly editorial group organized a technical conference called *Web 2.0*. At that moment in time the combination of diverse points, among which we have to cite the widespread adoption of broadband internet access to the homes, led to renewed interest on world wide web technologies. Since then we speak of "web 2.0 technologies" when talking about a set of relatively new ways of doing things on the web. As the internet has become a medium for most activities, this "web 2.0" (O'Reilly, 2005) has had its effect on almost all of them, and now we talk, for example of business 2.0 and enterprise 2.0... and also of education 2.0. Essentially, we may speak about 'web 2.0' when:

- we consider the web as a platform where software applications, rather than documents, live;
- these software applications are designed to take profit from "collective intelligence" and effectively move from a developer-centric point of view to a user-centred one;
- according to that user-centred model, the user's data is the most important element in every transaction, and developers allow users total or almost total control of their assets;
- as applications live on a web server, and not on a desktop, the meaning of software releases and versions is lost, because there is no need to deploy a new release of any application to have it working for every single user.

One very meaningful cause/effect of web 2.0 is the extreme lowering of the cost of access to very sophisticated resources to have a strong presence on the web. The advent of lightweight content management software, from personal blogging tools to corporate portal management systems allows any average web user or organization to publish quality information in a comfortable and efficient way. For educators this means that publishing content and using a new set of tools to communicate and work on a new medium has become possible. But the availability of these new tools presents new challenges: when should an educator use one of them? Which is better for each finality?

1.2 Pedagogical context

Briefly reviewing the theories of the pedagogical models behind traditional learning systems in chronological order, an evolutionary path can be established, that would lead us to the next stage: e-learning 2.0, starting with behaviorism, going through cognitivism and constructivism and leading to social constructivism (Downes, 2005). These theories, however, were developed before learning was impacted by ICT technologies. For the present situation Siemens et al. (2005) propose a new theory, "Connectivism", intended to explain both individual and social learning processes, going beyond the social constructivism approach. Its starting point is that knowledge exists by itself and "individuals are supposed to realize that knowledge by connecting the nodes where it's located; being that nodes other individuals, organizations, different clusters weakly tied. It is changing the know-how and know-what for know-where the knowledge is". Some of connectivism key principles are:

- Learning is a constant building of a network
- Capacity to learn more is more important than current knowledge
- Connections, not content, are the starting point of the learning process
- Knowledge can rest within the network, not only within individuals

The connection between these two facets or contexts of the same emergence process (connectivism and web 2.0) is the learning 2.0 paradigm. Some of its key principles are:

- Learner-centered design: the learner constructs her own knowledge
- From communities of practice to social networking: the learner shares her knowledge
- Teachers and learners as peers within a social network
- From traditional learning applications to open learning environments.

2. A case experience at the UOC

2.1 The UOC

The Open University of Catalonia[†] (UOC) is an exclusively on-line university born off the knowledge society whose mission is the facilitation of continuous learning. To this end it makes intensive use of information and communication technologies to offer education beyond time and space frontiers. Both the date of its foundation, 1994, and the high number of members in its community (about 37,000, including students from Spain and Latin America, professors, managers and other staff), make UOC a pioneer university in the field of higher and continuous education via the Internet.

The UOC educational model places the student as an active agent at the center of her learning process. Along this process, in each subject the student is assisted by a team of professors (both faculty and consulting professors). Furthermore, a tutor guides and advises the student from the moment she starts her degree until the moment she finishes it. The student is also helped by her partners in the virtual classroom spaces, which are located inside the UOC Virtual Campus (a high-developed intranet which supports all academic necessities, including registering and technical assistance). In these virtual spaces, the student can find a complete course syllabus, the course learning materials and other resources (as the Digital Library or academic software). The Virtual Campus also includes an e-mail service, a chat service and some other spaces (newsgroups and distribution lists) conveniently designed to promote interaction among students and among students and professors.

Learning and teaching in this virtual environment has forced the institution to overcome difficulties where no traditional models could help, and to invent new ways by using technology. Particularly, the teaching of higher level mathematics has meant a series of difficulties, as the impossibility of easily writing with mathematical notation by using a traditional keyboard or the loneliness in which our students faced the static didactic material (in paper or digital format). But these inconveniences have also been opportunities for the development of new tools and methodologies.

2.2 An experience of interactive calculus web materials

In this article we present one of these experiences, an interactive, web based learning material designed for the calculus course for the technical computer engineering studies.

This material uses the Wiris[‡] software, an on-line Computer Algebra System, which allows mathematical calculations on-line. This software was chosen because it offered some features that made it more suitable to our necessities than other existent commercial software, in particular:

- Its availability of both on-line and local versions, so students should be able to work from any computer connected to Internet, which is one of the main features of UOC.
- It is a multilingual tool that allows mathematics computing to be done in the students' mother tongue (Spanish or Catalan in our case).
- It has very intuitive commands and the screen resembles closely a blackboard or the sheet of paper where a student could write.
- It allows us construction of interactive learning exercises.

The learning material we are going to present, although impregnated with the use of Wiris, was designed and created to be used with any other software of similar features. What is essential is

[†] UOC, http://www.uoc.edu
[‡] Maths for More http://www.mathsformore.com/

the integration of mathematical software, not any particular software. In this sense, this experience must be set in the methodological model followed and whose key issues are the following ones:

- Preference must be given to learning concepts over learning calculation algorithms.
- Prefererce is also givent to learn mathematical process over memoraizing content.
  - Problem-solving.
  - Mathematical modelling.
  - Reasoning.
  - Communicating.
  - Making connections (applying maths).
  - Using tools.
- Formalism and algorithmic have to be softened without losing mathematical rigour.
- Mathematical software must be intuitive and multilingual.
- There is a need for a continuous evaluation process.
- Special attention must be paid to the determination of each student's background.

2.3 Web based learning materials

After deciding that mathematical materials should be put on the web, one should consider the different available formats.

Perhaps the most widespread tool to export mathematical notation to the web is the one used in Wikipedia, a program called Texvc[§], able to transform standard mathematical LaTex notation into PNG graphic files. Other tools working in a similar way are TeX4ht[**], Hermes[††], TtM[‡‡] and blahTeX[§§]. They all work in approximately the same way: they take the standard text and convert it into HTML and/or XHTML, and mathematical notation is converted into either a graphic file (usually in the PNG or GIF formats) or MathML mark-up.

Thinking about "forward-compatibility", and worrying about the inevitable loss of information when converting mathematical mark-up into images, we chose the tex4moz LaTeX package and the option to generate an XHTML plus MathML combination. While graphic support is already universal on every browser, modern browsers on every computing platform support MathML: Firefox does support it natively since the 1.5 release (depending on the operating system, some additional typography downloads might be necessary), and support for Internet Explorer is in the form of a freely available plug-in[***].

The conversion process is not trivial: tex4moz is still in development, and while it works remarkably well, it can't be expected to know how to translate every single possible combination of mark-up in a LaTeX document, specially if non-standard packages are used. While the package hasn't reached a more advanced level of evolution, users should be advised to try and keep the use of packages to the bare minimum, and to try to adhere to the rules as strictly as possible. Any error in the generated XML document is hard to correct, because it makes the document non-valid, so browsers won't render it, providing no visual feedback about the nature of the error.

[§] Texvc, http://en.wikipedia.org/wiki/Texvc
[**] TeX4ht: LaTeX and TeX for Hypertext, http://www.cse.ohio-state.edu/%7Egurari/TeX4ht/
[††] Hermes - a semantic XML e-publishing tool for LaTeX authored scientific articles, http://hermes.roua.org/
[‡‡] TtM, a TeX to MathML translator, http://hutchinson.belmont.ma.us/tth/mml/
[§§] Blahtex, http://www.blahtex.org/index.php?page=home
[***] MathPlayer, http://www.dessci.com/en/products/mathplayer/

When editing MathML mark-up, a very useful tool is Amaya[†††], a W3C tool providing a visual equation editor.

From a developer's point of view, Wiris takes the form of a Java applet taking its parameters in the form of MathML commands. Neither the developer nor the students need to have any knowledge of MathML, as the applet provides an easy to use editor and menu based tools.

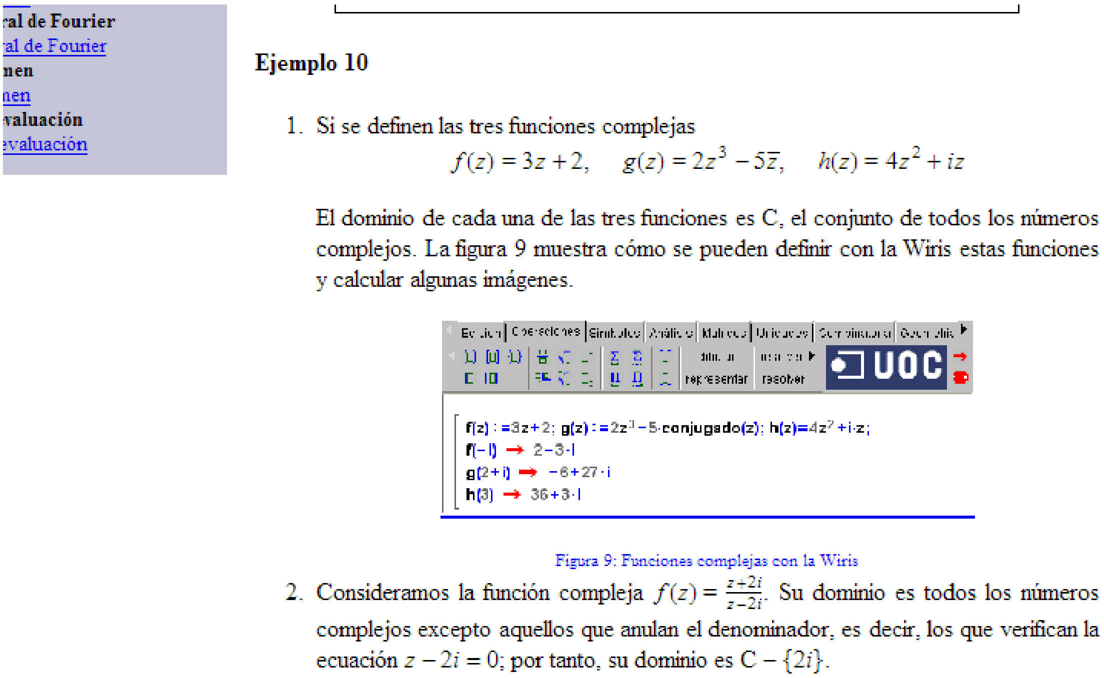

Fig. 1. Example of the UOC-Spanish language version of the web based material

2.4 Results

2.4.1 Format

As a result we have obtained a simple interface, with acceptable mathematical notation and easy navigation. But there is more: where in a traditional material we would have an exercise, now we have a link to some interactive material developed with Wiris providing an infinite number of exercises. Where in a traditional book we would have a figure and a written explanation, here we can offer the possibility of a dynamic experiment to practise the concepts.

2.4.2 Visualization and interactivity

Activities created with the symbolic calculator Wiris were incorporated into it. There were two main goals to this:

- Complementing the software's documentation and indicating at every step the most efficient use of the included resources
- Presenting interactive learning activities, elaborated in the Wiris environment in order to reinforce concepts. It is here that the web presentation of the materials gives an added value over paper based materials.

The fact that Wiris includes its own programming language, structured and with an easy syntax (with the relativity we always have to use when talking about "easiness" when we speak about programming languages) allows to fulfill another goal, a "subliminal" one, we could say:

[†††] Amaya Home Page, http://www.w3.org/Amaya/

attending to the fact that the materials belong to a computing engineering subject, could we get students to try to write on their own small programs of mathematical application in subjects related with the subject at hand? Even if this was not our main goal, through the consultations received in UOC's virtual campus, we can assure it has been the case repeatedly.

In order to work on calculus operating resources in the context of the teaching material in web format, the easiness with which Wiris is able to present an embedded active screen in an HTML page is very interesting, as it puts all of its features within the user's reach. In this way, what in paper based materials appears as a screen capture where the suitable commands are explained becomes now interactive. The work elements can be changed, and the corresponding change in the result observed. And then new calculations can be introduced because the user is free to experiment on his own.

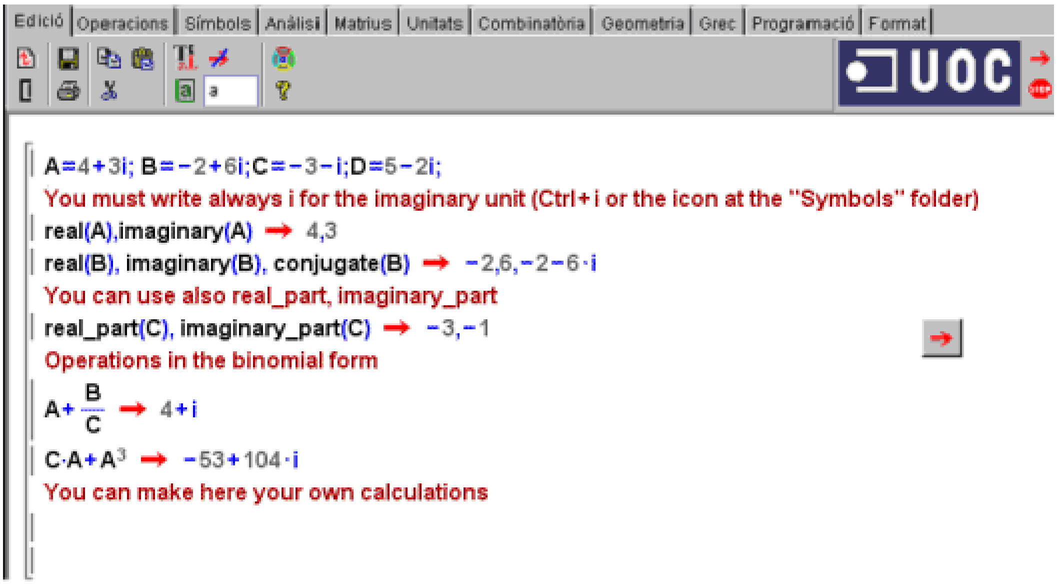

Fig. 2. The first screen of calculations with complex numbers. The screen capture corresponds to the UOC-Catalan language version, with comments rewritten, this time, in English

Certainly, this change from *illustrative images on paper* towards *active work screens* in the material is interesting for users. Nonetheless, the moment where an undoubtedly added value over paper based material becomes obvious where graphics become interactive, but with an active interactivity: the user cannot become a simple spectator, and he has to take decisions about how he wants to make the study evolve.

Wiris graphics are generated in a graphic board that opens as a pop-up window over the screen we are working on. User can move points used as a basis to generate the graphic and, in that way, experience new situations and variations of the original problem. The fact that the graphic board is not embedded in the page and has to be activated on the user's petition —something that could be considered a limitation— becomes interesting because page load is faster but, mainly because in this way the graphics window size can be manually modified, not to zoom in (although that can be done too, of course) but to observe with the same detail level other zones in the drawing that might be invisible originally.

To end this section, we will show an image of one of the activities included in the materials.

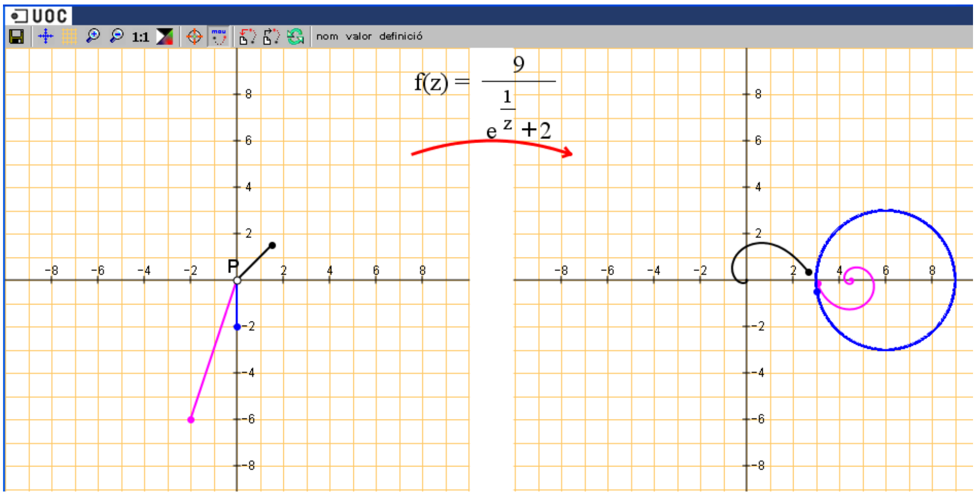

Fig .5. A complex function without limit for z=0. The trajectories approaching z=0 and their images are shown in an interactive way. Both the studied point and the approaching trajectories can be changed interactively.

There is no doubt that being able to interact in a presentation like this one helps understand the concept of the limit on a point of a complex function.

3. Goals & Conclusion

This experience was positive although it could be improved. Students are having scarce use of the web material, due to the fact that they try to do the minimum work necessary to pass the course, that is, to pass the exam. As a way to improve the use of computer algebra systems and interactivity activities may aspects of the pedagogic approach have still to be done:

- New ways to teach (technologies) demand new things to teach. Thus, rethink the math curriculum and its methodology is needed.
- Also important is to have usable new tools (design and interface)
- A modification in the general learning strategy should be helpfully: learning to learn is more important than learning contents
- A change of the assessments and certification systems are necessaries when dealing when that new technologies.

As a conclusion, the fact that currently there are computer algebra systems within reach of every student must necessarily change teaching habits and methodologies: from the focus of teaching materials to the very own goals of teaching procedures and teaching to learn. The situation could be analog to what happened a quarter of a century ago with the introduction of scientific calculators in the classroom.

We also have to note that access to information through he Internet makes everything more accelerated. The debate over the subject has also fastened its pace. In the elaboration of these materials we have commented on, we have firmly decided in favour of an option, intending to achieve an evolution from memorizing concepts and algorithms to understanding concepts and the use of those algorithms. We believe we have made some advance in that path.

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