A hybrid algorithm combining metaheuristic with Monte Carlo simulation for solving the Stochastic Flow Shop problem

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Abstract

In this paper, a hybrid simulation-based algorithm is proposed for the Stochastic Flow Shop Problem. The main idea of the methodology is to transform the stochastic problem into a deterministic problem and then apply simulation to the latter. In order to achieve this goal, we rely on Monte Carlo Simulation and a adapted version of a deterministic heuristic.

This approach aims to provide flexibility and simplicity due to the fact that it is not constrained by any previous assumption and relies in well-tested heuristics.

Keywords: *Stochastic Flow Shop Problem, metaheuristic, hybrid algorithms, Monte Carlo simulation*

1. Introduction

In this paper, a hybrid simulation-based algorithm is proposed for the Stochastic Flow Shop Problem (SFSP), a variation of the Flow Shop Problem (FSP). The FSP is a well-known scheduling problem in which a set of independent jobs with random processing times have to be executed in a set of machines in a determined sequence. There are some costs associated with the running of theses jobs that could be considered, such as preemption o allocation costs. The classical goal is to determine the sequence or sequences with the best running performance. In this non-deterministic version of the Flow Shop Problem, the time needed for each job for being executed is unknown and follows certain probability distribution. So, the proposed method will treat to expose a new hybrid algorithm which aims to be as adaptive and efficient at least as the works to date.

The study of the SFSP is within the current popularity of introducing randomness into combinatorial problems. It allows to describing new problems in new realistic scenarios where some information cannot be obtained previously.

It is important to remark the FSP as a relevant topic for current research, so a large set of efficient optimization, heuristic and metaheuristic methods have been developed. As it has happened with other combinatorial problems, a large number of different approaches and methodologies have been developed to deal with it. These approaches range from pure optimization methods (such as linear programming) for dealing with small-sized problems to use of heuristic and metaheuristics to provide near-optimal solutions for medium and large-sized problems. Usually, the main criterion to minimize is the makespan, but other criteria different to total time employed to process the list of jobs should also be considered in order to provide the decision-maker a large set of near-optimal and efficiently obtained options where he/she could choose according to specific necessities.

The main difference between the FSP and the SFSP is that in the former job processing times are known beforehand, while in the latter the actual processing time of each job has a stochastic nature, i.e. the statistical distribution is known beforehand, but its exact value is revealed only when the job is to be launched. So, while in the deterministic problem the performance measure is usually the makespan, in the non deterministic version this measure is not representative. This is due to the fact that we do not know the execution times of the jobs involved, so it has no sense to talk in terms of concrete values but in terms of probability. So, we must consider the expected makespan to have a valid way of measuring the performance. As a result, our final target is to minimize the expected makespan of the obtained sequences.
Besides, there are a large set of methods developed for the FSP. This is not the case of the SFSP. These methods need to be efficient to provide near optimal solutions to small and medium instances of the problem in a reasonable time.

The rest of this work is structured as follows: in section 2 the SFSP is presented with a detailed formulation and section 3 provides a literature review in order to expose an overview of the state of the art. In section 4 the main ideas of our approach are explained, and section 5 discusses the advantages offered by our methodology over the existing to date. Section 6 shows numerical experiments that illustrate our methodology, with a discussion of its results in the Section 7.

Section 8 describes future work to perform on related topics. Finally, section 9 summarizes the main contributions of this paper.

2. Basic notation and assumptions

The SFSP is a well-known scheduling problem that can be formally described as follows: a set J of n independent jobs have to be processed on a set M of m independent machines. Each job $j \in J$ requires a stochastic processing time p_{ij} n every machine $i \in M$. This stochastic processing time is defined by a certain distribution (e.g. normal, exponential, triangular, etc.). The classical goal is to find a sequence for processing the jobs so a given criterion is optimized. The most common used criterion is the minimization of the maximum completion time or makespan, denoted by C_{max}.

The described problem is usually denoted as $Fm|prmu|C_{max}$, which means that it is a flow-shop permutation problem with m machines with the aim to minimize the makespan.

In addition to this definitions, it is assumed that:

- all the jobs are processed in all the machines in the same order.
- there is unlimited storage between the machines, and non-preemption.
- machines are always available.
- each machine can process only one job a time.
- a job cannot be processed more than once for machine.
- job processing times are independent and exponentially distributed variables.

At this point, it is interesting to notice that our approach does not require to assume any concrete distribution for the stochastic variable whereas it is in most previous stochastic flow shop approaches.

3. State of the art and related work

The FSP is a NP-complete problem (Rinooy kan 1976) which consists, as mentioned, in running a set of J jobs in a set of M machines with the goal of minimizing total execution time or makespan. We focus on the stochastic version, where job-processing times follow probability distributions. The first publication about flow shop scheduling with random processing times appears in Markino (1965), where a problem with two machines and two jobs is presented.

Many works have focused on the importance of considering uncertainty in real world problems, specifically in the case of scheduling-related problems. Thus, Al-Fawzan et al. (2005) analyzes the Resource Constrained Project Scheduling Problem (RCPSP) focusing on makespan reduction and robustness. Jensen (2001) also introduces the concepts of neighborhood-based robustness and tardiness minimization. Ke (2005) proposes a mathematical model for achieving a formal specification of the Project Scheduling Problem.

Allaoui (2006) studied makespan minimization and robustness related to the SFSP, suggesting how to measure the robustness. Proactive and reactive scheduling are also characterized in his work. An example of reactive scheduling can be found on Honkonp (1997), where performance is evaluated using several methodologies. On the other hand, robustness in proactive scheduling is analyzed in Ghezail et al. (2010), who propose a graphical representation of the solution in order to evaluate obtained schedules.

As the concept of minimum makespan from FSP is not representative for the stochastic problem, Dodin (2006) proposes an optimally index to study the efficiency of the SFSP solutions. The boundaries of the expected makespan are also analyzed mathematically.

A theoretical analysis of performance evaluation based on markovian models is performed in Gougand et al. (2005), where a method to compute expected time for a sequence using performance evaluation is proposed. A study of the impact of introducing different types of buffering among jobs is also provided in this work.

On the other hand, Integer and linear programming has been employed together with probability distributions to represent the problem in Janak 2007.

Many heuristics and metaheuristics have been proposed in order to solve the PFSP due to the impossibility of finding exact solutions efficiently. For example, Johnson (1954) suggested a simple heuristic to solve the two machines problem, while Campbell et al. (1970) built a heuristic for PFSP with more than two machines.

NEH is considered the best performing heuristic, it was introduced by Nawaz et al. (1983). Later, Tailard (1990) reduced NEH complexity by introducing a data structure to avoid the calculation of the makespan.

Moreover, Ruiz (2005) proposed an Iterated Greedy (IG) algorithm for the PFSP built on a two-step methodology.

Some successful works have been published about dealing with indeterminacy in other combinatorial problems as the Vehicle Routing Problem (VRP), with Stochastic Demands (VRPSD). The VRPSD is a variation of the VRP, where the demands to serve are unknown and follow certain probability distributions.

In some recent works, Juan et al. (2009a, 2009b, 2010a) describes the application of simulation technologies to solve Vehicle Routing Problems. Thus, in Juan et al. (2011), the stochastic problem is transformed into a deterministic instance using the concept of safety-stocks as a confidence margin over the variance of the demands. This subsequent deterministic problem is solved employing proven efficient heuristics. Then, simulation is applied in order to evaluate the probability of failure of the obtained sequences.

Simulation has been applied in the SS-GNEH (Juan et al. 2010b) to solve the FSP in , where the NEH algorithm is modified to have some random behavior building a GRASP-like methodology.

In the stochastic variant, simulation based approaches usually rely on performance evaluation as in Gougard (2003). Dodin (2006) performs simulations to prove its empirical analysis of the boundaries of the makespan.

Moreover, Honkonp (1997) relies on the use of simulation for reactive scheduling while Ghezail et al. (2010) graphical representation that has been mentioned previously.

4. Proposed methodology

The main idea behind the methodology is that the initial stochastic problem (SFSP) will be transformed into a deterministic version (FSP), where well-known heuristics exist. Because well-tested meta-heuristics exists for the FSP, such as SS-GNEH, we will apply it in order to get some efficient solutions for this deterministic problem.

The transformation is achieved by assigning the expected value associated to the probability distribution that described the stochastic job processing time, to the deterministic.

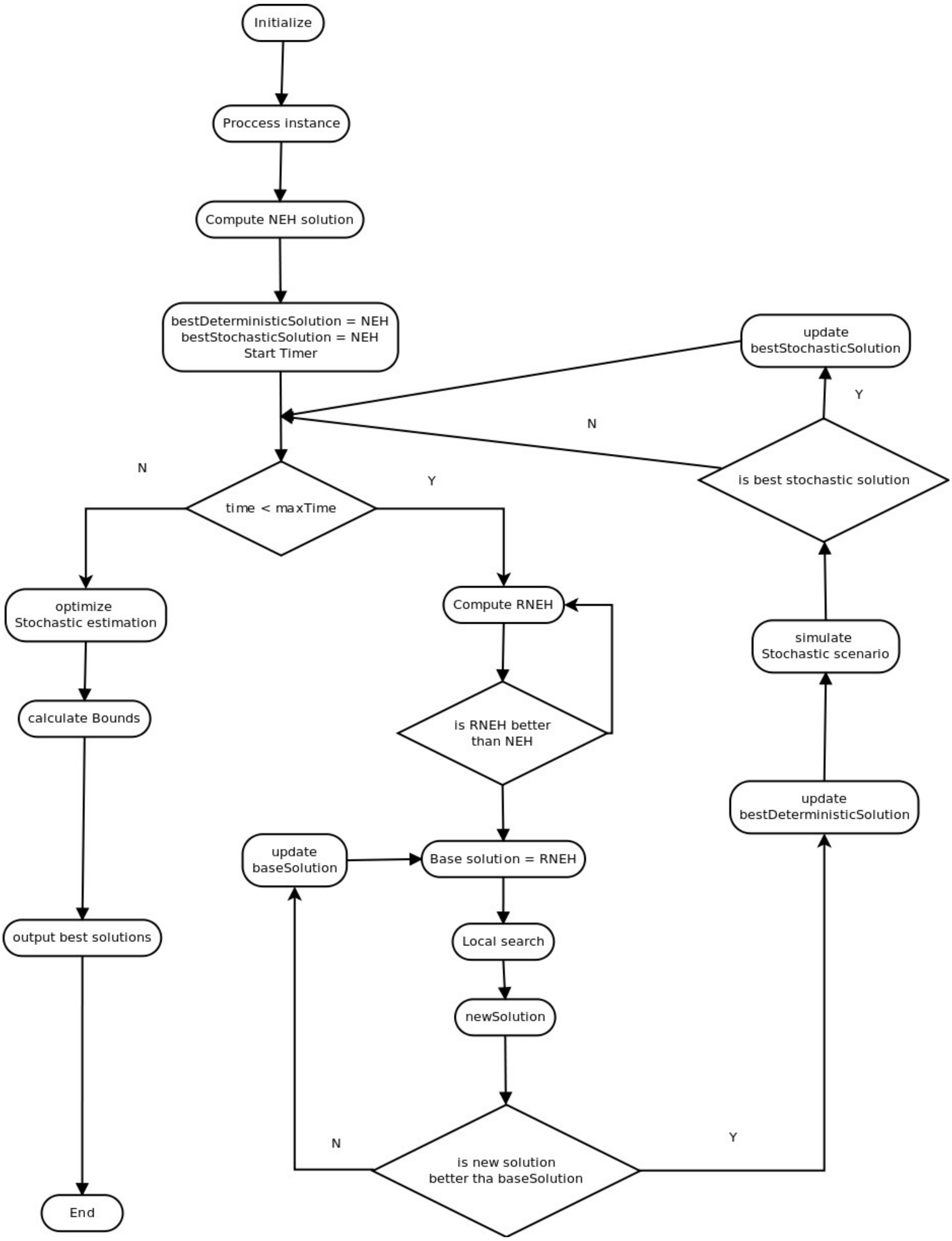
As any feasible solution for the deterministic instance is also a potential solution for the SFSP, we can extrapolate it to the stochastic scenario. Then, we estimate the expected makespan for the stochastic problem by using the direct Monte Carlo method. So the obtained solution will be simulate in the probabilistic scenario. The simulation will be run as many times as we need to get an estimation reliable enough.

The specific steps describing the proposed methodology are the following:

1. Consider a SFSP instance defined by a set J of jobs and a set of M machines with stochastic processing times for each job in each machine.

2. Consider for each job with processing time p_{ij} the expected value for the corresponding probability distribution: $p^*_{ij} = E[p_{ij}]$.
3. Let be FSP* the resulting problem defined by the deterministic demands p^*_{ij} .
4. Generate some efficient random solutions for the FSP* by applying a proven heuristic such as SS-GNEH.
5. For each feasible solution generated, run some simulations in order to obtain an estimation of the expected makespan (the more simulations we perform, the more exact estimation is obtained).
6. Store the best stochastic and deterministic solutions
7. Calculate a more exact expected makespan estimation for both solutions
8. Test that the stochastic solution is the best, change for the deterministic otherwise
9. Output the solution

The graphical representation of this process is as follows:



4.1 Algorithm Pseudo-code

The pseudo-code 1 shows the main procedure of our test execution, which drives the methodology.

```
procedure Main
for each instance from test2run
begin
  solution = StochasticSSGNEH.solve(instance)
end
lowerBound = getBestDeterministicSolution()
SimulateStochastic(GetBestDeterministicSolution(), 100000) //Compute
Upper-bound
getBestStochasticSolution()
ShowSolutions
```

First inputs are entered in the program and each test is run for solving each problem instance using proposed methodology. After that, the lower and upper bounds are calculated in order to test the efficiency of the obtained solution. At last, solution is provided to the output.

```
StochasticSSNGEH.solve(instance)
  //For each feasible solution generated during the SSGNEH calculus
  solution = SSGNEH.solve(instance) // Deterministic algorithm
  solution.stochasticCost = getStochasticCost(solution, 1000)
  storeBestDeterministicSolution(solution) //Current best
deterministic solution
  storeBestStochasticSolution(solution) //Current best stochastic
solution
  improveEstimation()
return solution
```

Pseudo-code 2 offers the main procedure of our methodology, where the stochastic cost is estimated for each feasible solution obtained by the SS-GNEH when solving the deterministic problem. We run the necessary simulations in order to obtain a first and efficient approximation of this cost. Once the cost is estimated it is compared to the cost of the best stored solution, if it is better, we choose this latter solution as candidate. This steps will be performed for all the solutions provided by the deterministic problem associated.

Pseudo-code 3 provides the details of the Monte Carlo simulation of the stochastic scenario. Here, we will iterate over all jobs and all machines to construct the distribution associated to each permutation. To do that we will rely on the SSJ library for stochastic simulation. Next, we generate random values from the distribution for constructing a probabilistic scenario.

Once we have obtained values for all pairs of job and machines we calculate the cost of the given solution and, after running as many simulations as we the number we passed in parameters we return the mean from all values.

```
getStochasticCost(solution, simulations)
for i from 1 to simulations do
begin
  for each job in solution.jobs
  begin
    for each machine in solution.machines
    begin
      dist = ExponentialDistributionFromMean(solution.cost(job,
machine))
      ExponentialGen gen = new ExponentialGen(dist)
      stochasticTimes.cost(job, machine) = dist.nextDouble
    end
  end
  makespan = calcost(stochasticTimes) //calcost is the same cost
calculation algorithm adapted to double values
  makespanSum = makespanSum + makespan;
end
return makespanSum/simulations
```

Pseudo-code 4 exemplifies the way of improving the solution quality. First we run stochastic simulations both for the best deterministic solution and the best stochastic solution stored. We do that as many times as we need to get a estimation reliable enough. Finally, we compare the expected makespans to test that the proposed best solution stays between the bounds. If it is no so, we correct it.

```
improveEstimation()
 if (SimulateStochastic(GetBestDeterministicSolution(), 100000) <
SimulateStochastic(GetBestStochasticcSolution())
then
   storeBestStochasticSolution(GetBestDeterministicSolution())
 end
```

5. Advantages and contribution of our approach

Despite the idea of solving a stochastic problem by solving a related deterministic problem is not completely new (see the state of art section), it has not been developed for solving the SFSP. In fact, most of the works to date have focused in the theoretical aspects of the stochastic scheduling. By contrast, the proposed method provides a practical approach to the solution, with some potential benefits.

The potential benefits of our approach are:

- While previous approaches rely on theoretical chance-constrained models, our approach suggests a more practical perspective. This allows us to deal with more realistic scenarios.

- By using Monte Carlo simulation in our methodology, it allows to build models based in different random variables while previous approaches use to assume a particular behavior. More exactly, any probability distribution for describing the job processing times could be implemented with no restriction.

Thus, as far as we know, the presented methodology offers some unique advantages over the existing to date for solving the SFSP. In fact, some of these potential benefits are:

- The methodology is valid for any statistical distribution with a known mean, both theoretical -e.g. Normal, Log-normal, Weibull, Gamma, etc.- and experimental.
- The methodology reduces the complexity of the SFSP -where no efficient methods are known yet- to the FSP, where mature and extensively tested heuristics have been developed. This increases the credibility of the provided final solution.
- Because it is based on simulation, the methodology can be extended to consider a different distribution for each job processing time, possible dependencies among jobs, etc.
- The methodology can be applied to SFSP instances of virtually any size, so complexity attached to size can be managed by efficient SFSP metaheuristics and Monte Carlo simulation.

In summary, the benefits provided by our methodology can be summarized in two points: simplicity and flexibility. It is simple because is parameter-free and does not need fine tunning and flexible because it can be adapted to other probabilistic scenarios of virtually any size or admit more variables.

6. A numerical experiment

Described methodology has been implemented as a Java application. Java language was selected because of being an object oriented language that facilitates the rapid development of a prototype. A standard personal computer Intel(c) 7i CPU a t 2.6 GHz and 3 GB RAM was used to perform all test, which where run directly from the Eclipse IDE.

There is a lack of specific benchmarks for the SFSP, this could be due to the mentioned fact that most works about the problem have been mainly theoretical. So we have decided to use a benchmark created for the deterministic FSP.

In order to test our heuristic with that deterministic test we assume that, in each test instance, the processing time for each job is considered as the mean of the probabilistic distribution associated to that job in the stochastic case. After that, we generate random values based on the resulting distribution function.

We have tested 90 instances from Taillard tests (1993) with different seeds and we have generate probabilistic values based on triangular distributions. For each instance, we have considerate the solution given by the NEH heuristic, the best solution obtained with the SS-GNEH heuristic for the deterministic case and the best solution obtained for the deterministic scenario.
In addition, to generate probabilistic values, we use a triangular distribution defined with the parameter $\lambda = 1/E(x)$, being E(x) the expected value. This distribution and the probabilistic generation of random values are implemented using the SSJ library for stochastic simulation that can be found in http://www.iro.umontreal.ca/~simardr/ssj/.

Before discussing the results of the methodology, we will focus in one instance of the tests to illustrate our methodology. Given tai001_20_5 instance from the Taillard tests with the following processing times:

Job	**Machine 1**	**Machine 2**	**Machine 3**	**Machine 4**	**Machine 5**
1	54	79	16	66	58
2	83	3	89	58	56
3	15	11	49	31	20
4	71	99	15	68	85
5	77	56	89	78	53
6	36	70	45	91	35
7	53	99	60	13	53
8	38	60	23	59	41
9	27	5	57	49	69
10	87	56	64	85	13
11	76	3	7	85	86
12	91	61	1	9	72
13	14	73	63	39	8
14	29	75	41	41	49
15	12	47	63	56	47
16	77	14	27	40	87
17	32	21	26	54	58
18	87	86	75	77	18
19	68	5	77	51	68

20	94	77	40	31	28

First, we solve this deterministic instance using the SS-GNEH algorithm, in the first iteration we obtain the permutation of jobs:

```
3,15,6,19,14,9,17,18,5,7,8,16,4,11,13,1,2,10,20,12
```

With the associated cost 1279.

After that, we convert it to a stochastic case by considering these processing times as the mean of associated probabilistic distribution functions. So, we obtain the following stochastic instance:

Job	**Machine 1**	**Machine 2**	**Machine 3**	**Machine 4**	**Machine 5**
1	54	79	16	66	58
2	83	3	89	58	56
3	15	11	49	31	20
4	71	99	15	68	85
5	77	56	89	78	53
6	36	70	45	91	35
7	53	99	60	13	53
8	38	60	23	59	41
9	27	5	57	49	69
10	87	56	64	85	13
11	76	3	7	85	86
12	91	61	1	9	72
13	14	73	63	39	8
14	29	75	41	41	49
15	12	47	63	56	47
16	77	14	27	40	87
17	32	21	26	54	58
18	87	86	75	77	18
19	68	5	77	51	68
20	94	77	40	31	28

Next, we apply Monte Carlo Simulation in order to obtain the expected makespan of this job ordination. To achieve that, we generate random processing times based on the constructed probability distribution (triangular distribution as it has been stated) obtaining the following values:

Job	Machine 1	Machine 2	Machine 3	Machine 4	Machine 5
1	53.95231664418864	9.554324859076363	4.059819828346991	21.437067621716718	8.349095460126442
2	10.851787267698429	91.46005676974944	35.83763885900035	146.08657218939328	18.461884218872605
3	3.0081763388665252	74.07889293605818	2.5904531998378104	387.37039921251807	16.076462561126057
4	46.192910414844	0.8149869032398733	40.636353639872894	40.69516755980336	122.12764384230167
5	94.56294692075898	1.1772624863448413	55.26016355122542	2.684559543218368	22.531359012907842
6	29.033858900786257	3.338727595017999	55.70749588616669	53.68022159140592	9.65154097601944
7	5.613540711704708	3.2676925492616085	68.65979300946832	76.28919310527436	123.77580750894643
8	133.1927192309539	15.179903613315757	11.996223935475877	19.538318354179047	13.294782802073815
9	26.18825455521346	47.5130744716464	160.84541592595713	85.44135801684438	44.74950855140445
10	68.97209894736396	53.43971928752163	69.87585745188214	3.446862216215241	98.97624594141396
11	20.53096435540511	23.430624201722324	1.5795915612903546	315.2886728493502	82.86828027001228
12	29.07121174850957	31.391331640945022	17.29774230643515	309.1558177705596	40.63992508499161
13	9.288702553954097	173.82359494129724	9.97665726653233	277.6099335942938	9.406954323175441
14	197.19307601188643	0.204653211418786	18.564442931524002	79.49458964837477	2.299912378718405
15	28.89857595517863	11.916628000834773	98.29127670351119	8.416441734299626	3.218076198605038
16	7.900175053759244	66.1247187007832	17.127595253307533	202.1729718573425	18.960013984275502
17	21.82758712783715	0.04000519724418818	101.56883829078609	62.580178844182704	14.068610008992977
18	143.4476201390987	139.36158099887038	34.154855080851895	94.31642586725393	5.449153189599154
19	41.83784434164883	0.41469544573021266	26.593006064417743	21.20422027895833	30.91827788124705
20	25.631890535301313	20.568620521375948	0.42614912899616536	11.570901789733627	110.6925145548281

Subsequently, we calculate the total cost of a problem with the obtained solution and generated processing times values to obtain the cost of `2519.1844962101795`. If we repeat this calculation 1000 times, we obtain a first approximation to the expected makespan: `1796.8382775738291`.
We store both deterministic and stochastic values (as we are in the first iteration it is the best solution until now) and run next iteration to obtain another feasible solution from SS-GNEH:

```
9,17,8,3,6,19,15,1,2,13,4,14,11,5,18,16,10,7,20,12
```

We follow the same procedure to compare the new obtained costs (a deterministic cost of 1286 and a stochastic cost of 1774.8112823703784) with the previous obtained values and we store the best solution both cases.

By performing this procedure 100000 times we are able to obtain an estimated best deterministic solution, and a estimated best stochastic solution. Now, a more exact value of the expected makespan of both solutions can be calculated by running previous simulation more number of times.

If we find that when the cost estimation is improved, the best stochastic solution cost is greater than the the best deterministic solution stochastic cost (which could be expected because of the nature of the random-based calculation), we correct it by taking the best deterministic solution as the the best stochastic one too.

At the end of the calculations, we would obtain the output with the best solutions found and its calculated and estimated costs:

```
*************************************************
*       RESULTS FROM SIMSCHEDULING PROJECT        *
*************************************************


-------------------------------------------
              NEH Solution
-------------------------------------------

Sol ID : 1
Sol deterministic costs: 1286
Sol time: 0h 0m 0s (0.002992901 sec.)
List of jobs:
3
17
9
8
15
14
11
16
13
19
6
4
5
18
1
2
10
7
20
12



-------------------------------------------
Our best deterministic solution (provided by the SS-GNEH)
-------------------------------------------

Sol ID : 8
Sol deterministic costs: 1278
Sol stochastic costs: 1770.877998309444
Sol time: 0h 0m 0s (0.172264997 sec.)
List of jobs:
9
3
17
6
15
14
11
13
5
4
19
8
18
7
16
```

7. Results

In this section, we will study the result obtained in the test simulation. Because we have run 15 executions of each instance with different seeds, we provide average values for each instance from all the executions.

In addition, we have presented them in a table that shows the following values:

- The number of the Taillard's test instance
- The jobs in the problem
- The machines in the problem
- The deterministic cost associated to the NEH solution
- The deterministic cost associated to the SS-GNEH solution as the average lower bound
- The average stochastic cost of the SS-GNEH solution as the upper bound.
- The average stochastic cost of the best stochastic solution
- The average gap of the stochastic solution with the lower bound.
- The average gap between the upper and the lower-bounds is provided in order to study the variance between these bounds.

Table 1 shows the results for the first 30 instances. These instances consider cases with 20 jobs.

Test	Jobs	Machines	NEH	Avg Lower bound	Avg Upper bound	Stochastic Cost	Avg Gap	Avg Bounds Gap
1	20	5	1286	1278.00	1784.96	1766.74	0.28%	0.40%
2	20	5	1365	1359.00	1864.06	1843.80	0.26%	0.37%
3	20	5	1140	1081.00	1561.79	1544.15	0.30%	0.44%
4	20	5	1325	1293.00	1875.93	1860.61	0.31%	0.45%
5	20	5	1305	1235.00	1731.05	1712.15	0.28%	0.40%
6	20	5	1228	1195.00	1697.49	1681.71	0.29%	0.42%
7	20	5	1278	1239.00	1746.46	1706.60	0.27%	0.41%
8	20	5	1223	1206.00	1756.02	1743.60	0.31%	0.46%
9	20	5	1291	1230.00	1761.61	1753.02	0.30%	0.43%
10	20	10	1151	1108.00	1605.16	1592.43	0.30%	0.45%
11	20	10	1680	1582.00	2423.04	2418.56	0.35%	0.53%
12	20	10	1729	1659.00	2573.74	2566.62	0.35%	0.55%
13	20	10	1557	1496.00	2344.98	2332.82	0.36%	0.57%
14	20	10	1439	1377.00	2126.28	2113.61	0.35%	0.54%
15	20	10	1502	1419.00	2217.42	2206.66	0.36%	0.56%
16	20	10	1453	1397.00	2148.26	2140.91	0.35%	0.54%
17	20	10	1562	1484.00	2239.79	2230.97	0.33%	0.51%
18	20	10	1609	1538.00	2372.59	2365.00	0.35%	0.54%
19	20	10	1647	1594.00	2439.49	2404.94	0.34%	0.53%
20	20	20	1653	1591.00	2482.14	2476.15	0.36%	0.56%
21	20	20	2410	2297.00	3595.50	3575.87	0.36%	0.57%

22	20	20	2150	2099.00	3298.44	3281.61	0.36%	0.57%
23	20	20	2411	2329.00	3639.60	3617.91	0.36%	0.56%
24	20	20	2262	2223.00	3460.68	3442.83	0.35%	0.56%
25	20	20	2397	2291.00	3602.36	3582.25	0.36%	0.57%
26	20	20	2349	2226.00	3479.58	3457.01	0.36%	0.56%
27	20	20	2362	2273.00	3557.57	3533.57	0.36%	0.57%
28	20	20	2249	2202.00	3454.92	3445.28	0.36%	0.57%
29	20	20	2306	2237.00	3504.33	3494.48	0.36%	0.57%
30	20	5	2277	2179.00	3387.19	3382.62	0.36%	0.55%

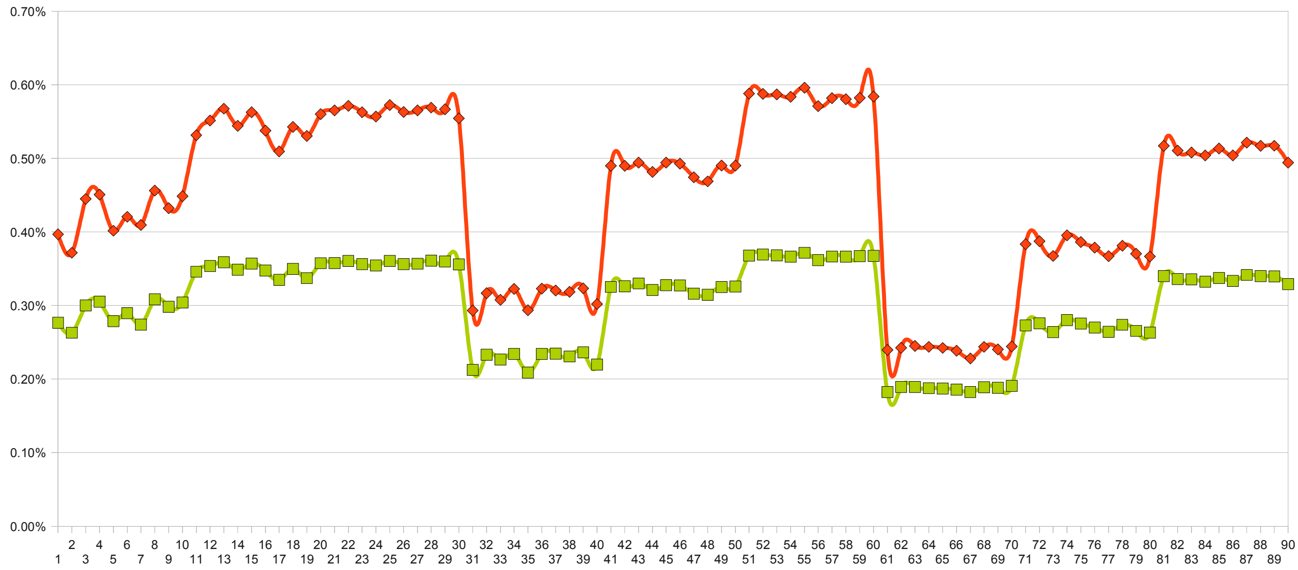
Table 2 shows the results for the instances 31 to 60. These are larger instances with 50 jobs. We can intuitively observe that the gap takes similar values as in the 30 first instances. Nevertheless, we can notice that the gaps are reduced for the first test with fewer machines.

Test	Jobs	Machines	NEH	Avg Lower bound	Avg Upper bound	Stochastic Cost	Avg Gap	Avg Bounds Gap
31	50	5	2733	2724.00	3522.70	3459.28	0.21%	0.29%
32	50	5	2843	2834.00	3731.96	3694.60	0.23%	0.32%
33	50	5	2625	2621.00	3427.64	3388.20	0.23%	0.31%
34	50	5	2782	2751.00	3638.33	3592.25	0.23%	0.32%
35	50	5	2868	2863.00	3703.07	3618.92	0.21%	0.29%
36	50	5	2835	2829.00	3742.09	3693.47	0.23%	0.32%
37	50	5	2736	2725.00	3597.94	3560.16	0.23%	0.32%
38	50	5	2690	2683.00	3537.44	3487.79	0.23%	0.32%
39	50	5	2571	2552.00	3377.34	3341.84	0.24%	0.32%
40	50	10	2786	2782.00	3622.38	3564.36	0.22%	0.30%
41	50	10	3136	3025.00	4506.76	4482.79	0.33%	0.49%
42	50	10	3021	2909.00	4334.14	4317.21	0.33%	0.49%
43	50	10	2952	2864.00	4279.23	4274.24	0.33%	0.49%
44	50	10	3183	3063.00	4537.90	4512.48	0.32%	0.48%
45	50	10	3128	2997.00	4478.06	4457.64	0.33%	0.49%
46	50	10	3158	3006.00	4487.17	4467.46	0.33%	0.49%
47	50	10	3277	3124.00	4605.84	4566.63	0.32%	0.47%
48	50	10	3123	3042.00	4467.61	4437.44	0.31%	0.47%
49	50	10	3002	2902.00	4324.26	4299.90	0.33%	0.49%
50	50	10	3257	3090.00	4604.51	4582.18	0.33%	0.49%
51	50	20	4013	3893.00	6182.12	6159.07	0.37%	0.59%
52	50	20	3921	3725.00	5914.06	5905.13	0.37%	0.59%
53	50	20	3890	3684.00	5846.34	5829.94	0.37%	0.59%
54	50	20	3926	3759.00	5952.62	5932.24	0.37%	0.58%
55	50	20	3822	3647.00	5820.71	5803.73	0.37%	0.60%
56	50	20	3914	3723.00	5848.38	5832.61	0.36%	0.57%
57	50	20	3952	3736.00	5909.25	5897.86	0.37%	0.58%
58	50	20	3916	3732.00	5898.40	5888.76	0.37%	0.58%
59	50	20	3952	3780.00	5980.83	5973.08	0.37%	0.58%
60	50	20	4016	3786.00	5996.64	5985.04	0.37%	0.58%

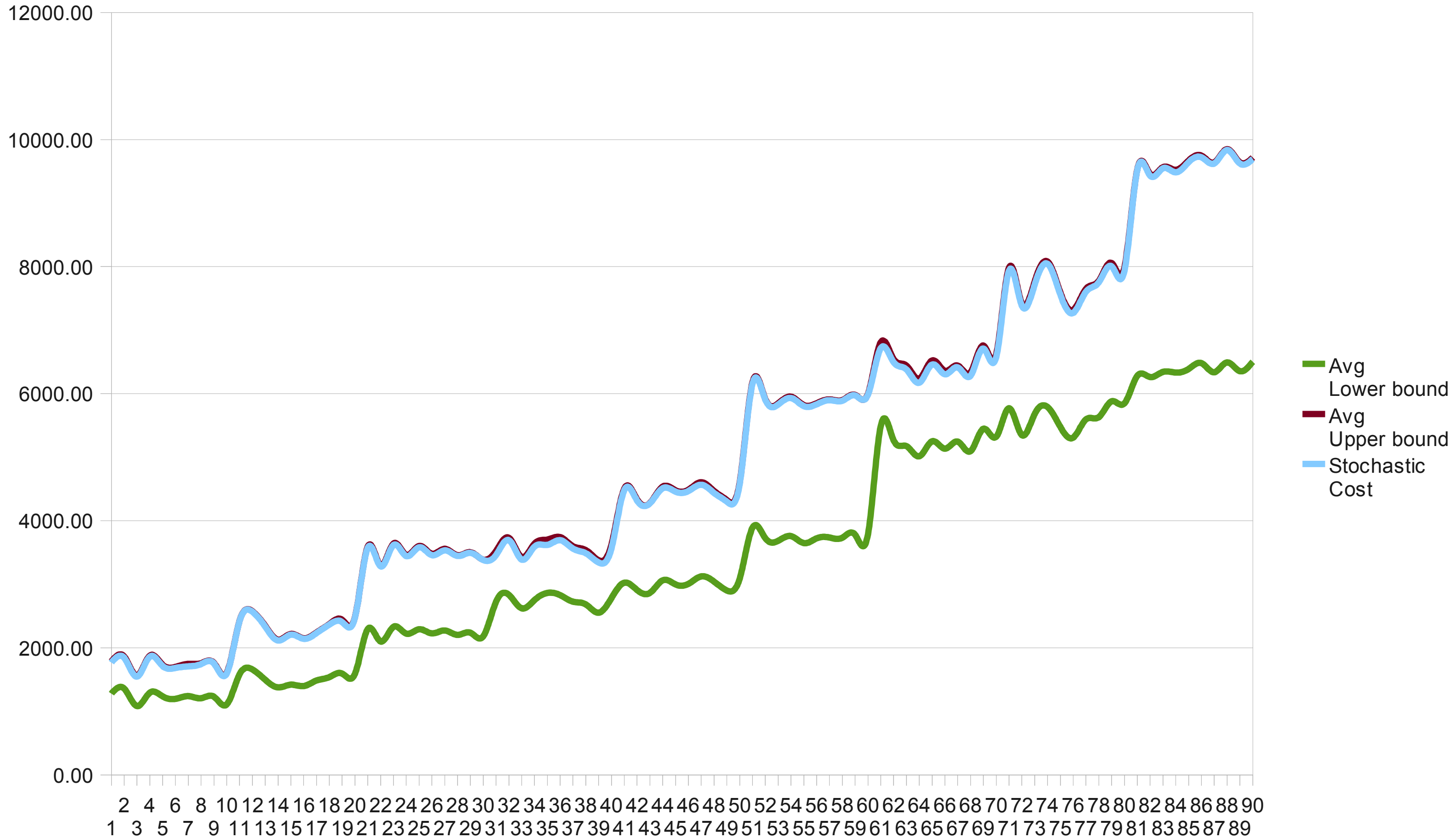
At last, table 3 shows the results for the instances 61 to 90. These are the largest instances with 100 jobs. We can see that, following the tendency suggested by the former table, the gaps keep reducing for low number of machines. In contrast, the gap increases for larger number of machines.

Test	Jobs	Machines	NEH	Avg Lower bound	Avg Upper bound	Stochastic Cost	Avg Gap	Avg Bounds Gap
61	100	5	5516	5493.00	6809.01	6717.21	0.18%	0.24%
62	100	5	5284	5268.00	6545.44	6496.53	0.19%	0.24%
63	100	5	5195	5175.00	6443.60	6382.25	0.19%	0.25%
64	100	5	5023	5014.00	6236.39	6171.95	0.19%	0.24%
65	100	5	5261	5250.00	6522.29	6458.81	0.19%	0.24%
66	100	5	5139	5135.00	6360.45	6305.33	0.19%	0.24%
67	100	5	5259	5246.00	6442.84	6415.35	0.18%	0.23%
68	100	5	5105	5094.00	6335.86	6281.29	0.19%	0.24%
69	100	5	5489	5448.00	6756.58	6710.72	0.19%	0.24%
70	100	5	5332	5322.00	6622.95	6576.24	0.19%	0.24%
71	100	10	5825	5771.00	7983.92	7937.18	0.27%	0.38%
72	100	10	5400	5349.00	7420.98	7386.03	0.28%	0.39%
73	100	10	5755	5679.00	7764.55	7715.57	0.26%	0.37%
74	100	10	5924	5791.00	8080.19	8045.43	0.28%	0.40%
75	100	10	5612	5478.00	7593.95	7560.71	0.28%	0.39%
76	100	10	5355	5308.00	7318.32	7271.01	0.27%	0.38%
77	100	10	5677	5596.00	7649.66	7605.66	0.26%	0.37%
78	100	10	5705	5630.00	7775.75	7752.52	0.27%	0.38%
79	100	10	5975	5880.00	8056.98	8005.56	0.27%	0.37%
80	100	10	5903	5848.00	7991.28	7936.59	0.26%	0.37%
81	100	20	6538	6281.00	9527.45	9517.08	0.34%	0.52%
82	100	20	6446	6263.00	9460.01	9432.26	0.34%	0.51%
83	100	20	6552	6343.00	9565.22	9548.23	0.34%	0.51%
84	100	20	6547	6333.00	9523.62	9485.02	0.33%	0.50%
85	100	20	6614	6389.00	9669.60	9643.86	0.34%	0.51%
86	100	20	6645	6481.00	9747.78	9722.86	0.33%	0.50%
87	100	20	6573	6337.00	9639.99	9623.97	0.34%	0.52%
88	100	20	6747	6493.00	9848.87	9835.94	0.34%	0.52%
89	100	20	6601	6355.00	9641.51	9623.66	0.34%	0.52%
90	100	20	6670	6503.00	9716.32	9694.36	0.33%	0.49%

In order to discuss these results, some visual support could be helpful. Thus, graph 1 provides the graphical representation of the calculated gaps, a quick view shows that best results are obtained in instances 31-40 and 61-70 where the number of machines is lower. We can observe that the gap seems to be greater the more machines there are. In contrast, it appears to generate quite similar values for same number of machines, no matter how many jobs are involved. So, it could suggest that this approach has potential power with problems with a large numbers of jobs.



On the other hand, graph 2, provides a graphical representation of the result comparatively with upper and lower-bounds. Here, we can clearly see that the obtained solution is always closer to the upper-bound than to the lower-bound. So, it seems to be some margin in order to improve the quality of the methodology.



8. Conclusions and future work

In this paper we have presented a probabilistic approach for solving the Stochastic Flow Shop Problem. This methodology combines Monte Carlo simulation with well tested methodologies for the Flow Shop Problem. The one of the basic ideas of our methodology is to decompose the SFSP into several FSP in order to obtain an estimation of the expected makespan for an efficient solution to the deterministic case by using simulation. This approach does not require any previous assumption and is feasible for any probabilistic function.

As a future work, some ideas are proposed. First, qualitative studies can be performed in order to compare robustness or different optimality measures. Next, the development of an approach which uses parallelization to perform the different scenarios of the simulation in different threads. We think this could improve performance and help to achieve more exact estimations of the expected makespan. The use of C/C++ versions of the code could reduce the computation times. Also, the study of the use of a security margin when transforming the stochastic problem into deterministic instances could improve the robustness and efficiency of the obtained solutions.

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