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Structural Reliability and Availability Analysis through Simulation

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Angel Juan Pérez; Jorge Simosa; Javier Faulin; Arai Monteforte; Harry Guo

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Abstract

In this paper, some potential applications of simulation in structural reliability and availability are presented. Unlike the analytical methods, where assumptions that simplify complex analysis often need to be made, simulation methods can model the structure reliability without simplifying the problem. We propose the use of statistical distributions and techniques –such as survival analysis– to model component-level reliability. Then, using failure- and repair-time distributions and information about the structural logical topology, structural reliability and availability information can be inferred through the use of discrete-event simulation techniques. Two numerical examples illustrate some potential applications of the proposed methodology.

1. Introduction

Structural Reliability & Availability (R&A) are critical issues in industrial engineering that have a daily impact on billions of people around the world, especially with the current trend towards globalization being enabled by structures such as bridges, airport towers, and government buildings. Historically, structural R&A has focused solely on methods for assessing the safety of civil and industrial engineering projects. However, with new emerging technologies such as wind and solar energy, structural reliability has a much broader range of applications that include the design and maintenance process of such structures. Some interesting references containing attractive cases with structural reliability problems are Blischke et al. (2000), Faulin et al. (2010), and Modarres et al. (2010).

Within the scope of structural R&A, a major concern of civil engineers is the ability to predict the lifecycle of structures that are exposed to stressful conditions such as constant workloads or natural disasters. Due to these exposures, the structures suffer degradation in the form of deterioration, deformation, fatigue, etc., resulting in possible environmental hazards. Thus, it must be noted that the state of the structures are persistently changing over time, as opposed to constant as suggested in other literature. According to Li (1995) there are three major ways in which structural concrete may deteriorate, namely: (1) surface deterioration of the concrete, (2) internal degradation of the concrete, and (3) corrosion of reinforcing steel in concrete. Of these, reinforcing steel corrosion is the most common form of deterioration in concrete structures and is the main target for the durability requirements pre-scribed in most design codes for concrete structures (Nilson et al., 2003). Overall, these aggressive conditions indicate that we should consider a structure's evolution over time when analyzing structural R&A.

The importance of using structural R&A to both design and maintain modern structures is highlighted by two recent examples of structural disasters, namely the Gulf Coast Oil Disaster (BP Oil Spill) and the accident in the Tibidabo (Barcelona, Spain) amusement park on July 17, 2010. The Gulf Coast Oil Disaster, regarded as one of America's worst environmental disasters, has resulted in being the largest accidental oil spill not in only the US, but in the world's history (Silverleib, 2010). The disaster was caused by an explosion on an offshore oil drilling rig that perhaps, through some careful structural R&A analysis, could have been prevented. Similarly, the accident in the Tibidabo, which resulted in one death and two others with severe injuries was the consequence of a component failure, a possible sign of ineffective maintenance policies.

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In this paper we propose the use of non-deterministic approaches – specifically those based on discrete-event simulation (DES) – as the most natural way to deal with uncertainties in time-dependent structural reliability and availability analysis. We begin by discussing how non-deterministic approaches differ from other approaches in the field and the benefits it may bring, such as higher accuracy in determining the reliability state of structures, which can be viewed as time-dependent systems through the consideration of time-dependent components and system topology. Our DES approach is then introduced along with two examples, namely a structural reliability case and a structural availability case. These examples illustrate just a few of the many applications that the DES approach offers within the structural R&A arena.

2. Structural Reliability and Existing Analysis Methods

Structural Reliability is an engineering discipline that provides a series of concepts, methods and tools to predict and/or determine the reliability, availability and safety of buildings, bridges, industrial plants, off-shore platforms and other structures, both during their design stage and during their operational lifecycle. As suggested by Melchers (1999), for any given structure, it is possible to define a set of limit states. These limit states represent varying levels of operative reliability and availability, which ranges from a fully operational state to a completely collapsed state. From a formal perspective, Structural Reliability is defined as the probability that a structure will not achieve each specified limit state –i.e. will not suffer a failure of certain type– during a specified period of time (Thoft-Christensen and Murotsu, 1986). The reliability or survival function is the probability that the structure will not have achieved the corresponding limit state at a given time. From a reliability point of view, one of the main targets of structural reliability is to provide an assembly of components which, when mounted together, will perform satisfactorily without suffering critical or relevant failures for some specified time period, either with or without maintenance policies.

In most cases, a structure can be viewed as a system of components (or individual elements) linked together by an underlying logical topology that describes the interactions and dependencies among the components. Each of these components deteriorates according to an analytical degradation or survival function and, therefore, the structural reliability is a function of each component's reliability function and the logical topology. Thus, it seems reasonable to assess the probability of failure of the structure based upon its elements' failure probability information (Mahadevan and Raghothamachar, 2000) (Coit, 2000). As described by Frangopol and Maute (2003), depending on the structure's topology, material behavior, statistical correlation, and

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variability in loads and strengths, the reliability of a structural system can be significantly different from the reliability of its components.

Therefore, the reliability of a structural system may be estimated at two levels: component level and system or structural level. At the component level, limit-state formulations, in addition to efficient analytical and simulation procedures, have been developed for reliability estimation (Park et al., 2004). In particular, a new structure will most likely have some components that have been used in other structural designs, from which there is an existing set of available data; on the other hand, if a new structure uses components about which no historical data exists, then survival analysis methods, such as accelerated life testing, can be used to obtain information about component reliability behavior (Meeker and Escobar, 1998). Structural-level analysis, on the other hand, addresses two types of issues: (1) multiple performance criteria or multiple structural states, and (2) multiple paths or sequences of individual component failures leading to overall structural failure. In structural level analysis, however, we must take into account any possible dependencies between components such as redundancy or reinforcement.

In general, structures must agree to a set of minimum design and construction standards, known as codes of practice, that correspond to the type of structure being designed. However, as noted by Lertwongkornkit et al. (2001), it is becoming increasingly common to design buildings and other civil infrastructure systems with an underlying "performance-based" objective which might consider more than just two structural states (collapsed or not collapsed); therefore, making it necessary to develop new techniques in order to account for uncertainty on key random variables affecting structural behavior. According to other authors (Marek et al., 1996) (Vukazich and Marek, 2001) standards for structural design are basically a summary of the current "state of knowledge" but offer only limited information about the real evolution of the structure through time. Therefore, these authors strongly recommend the use of probabilistic techniques, which require fewer assumptions, in order to deal with the uncertainties in structural design and decision-making such as how and when to perform maintenance on a structure. Camarinopoulos et al. (1999) do also recommend the use of probabilistic methods as a more rational approach to deal with safety problems in structural engineering. In their words, "these [probabilistic] methods provide basic tools for evaluating structural safety quantitatively". Moreover, Banks et al. (2009) emphasize the usefulness of simulation modeling for both predicting the effect of changes to existing systems and as a design tool to predict the performance of a system under varying sets of conditions.

As Park et al. (2004) suggest, it is difficult to calculate probabilities for each limitstate of a structural system. Structural reliability analysis can be performed using analytical methods or simulation-based methods (Mahadevan and Raghothamachar, 2000). On one hand, analytical methods tend to be complex and generally involve restrictive simplifying assumptions about structural behavior, which makes them difficult to apply in real scenarios. On the other hand, simulation-based methods can

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also incorporate realistic structural behavior (Billinton and Wang, 1999) (Marek et al., 1996) (Laumakis and Harlow, 2002). Traditionally, simulation-based methods have been considered to be computationally expensive, especially when dealing with highly reliable structures (Marquez, 2005). This is because when there is a low failure rate, a large number of simulations are needed in order to get accurate estimates –this is usually known as the "rare-event problem". Under these circumstances, the use of variance reduction techniques (such as importance sampling) is usually recommended. For an excellent review on simulation concepts, techniques, methods, and applications, the reader is referred to Banks et al. (2009), Law (2006), or Ross (2006). Nevertheless, in our opinion these computational concerns can now be considered mostly obsolete due to outstanding improvement in processing power experienced in recent years. This is especially true when the goal –as in our case– is to estimate time-dependent structural R&A functions, where the rare-event problem is not a major issue.

3. A Discrete-Event Based Approach

Consider a structure with several components which are connected together according to a known logical topology, a set of minimal paths describing combinations of components that must be operating in order to avoid a structural failure of some kind. Also assume that time-dependent reliability/availability functions are known at the component-level, i.e., each component failure- and/or repair- time distribution is known. As discussed before, this information might have been obtained from historical records or, alternatively, from survival analysis techniques -e.g. accelerated life tests- on individual components. Therefore, at any moment in time the structure will be in one of the following states: (1) perfect condition, i.e.: all components are in perfect condition and thus the structure is fully operational; (2) slight damage, i.e.: some components have experienced failures but this has not affected the structural operability in a significant way; (3) severe damage, i.e.: some components have failed and this has significantly limited the structural operability; and (4) collapsed, i.e.: some components have failed and this might imply structural collapse. Notice that, under these circumstances, there are three possible types of structural failures that can lead to a change in the state of the structure. Of course, the most relevant -and hopefully least frequent- of these structural failures is structural collapse, but sometimes it might also be interesting to be able to estimate the reliability or availability functions associated with other structural failures as well. To attain this goal, DES can be used to artificially generate a random sample of structural lifecycles, as seen in Figure 1.

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Figure 1: Using DES to generate a structural lifecycle

In effect, as explained by Faulin et al. (2008) component-level failure- and repairtime distributions can be used to randomly schedule component-level failures and repairs. Therefore, it is possible to track the current state of each individual component at each target time. This information is then combined with the structural logical topology to infer the structural state at each target time.

By repeating this process, a set of randomly generated lifecycles is provided for the given structure. Each of these lifecycles provides observations of the structural state at each target-time. Therefore, once a sufficient number of iterations have been run, accurate point and interval estimates can be calculated for the structural reliability at each target time (Juan and Vila, 2002). Also, additional information can be obtained from these runs, such as: which components are more likely to fail, which component failures are more likely to cause structural failures (failure criticality indices), which structural failures occur more frequently, etc. (Juan et al., 2007).

Moreover, notice that DES could also be employed to analyze different scenarios (what-if analysis), i.e.: to study the effects of a different logical topology on structural reliability, the effects of adding some redundant components on structural reliability, or even the effects of improving reliability of some individual components. Finally, DES also allows for considering the effect of dependencies among component failures and/or repairs. It is usually the case that a component failure or repair affects the failure or repair rate of other components. In other words, component failure- and repair-times are not independent in most real situations. Again, discrete-event simulation can handle this complexity by simply updating the failure- or repair takes place (Faulin et al., 2008). This way, dependencies can be also introduced in the model. Notice that this represents a major difference between our approach and other

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approaches –mainly analytical ones–, where dependencies among components, repair-times or multi-state structures are difficult to consider.

Thus, Figure 2 shows a step-by-step flowchart of our discrete-event based approach to structural reliability and availability problems.



Figure 2: Discrete-Event Based Methodology for Structural R&A Analysis

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4. Experiment 1: Structural Reliability

We present here a case study of two possible designs for a bridge. As can be seen in Figure 3, there is an original design (Case A) and an alternative with redundant components (Case B).



Figure 3: Different possible designs for a structure

Our first goal is to illustrate how a statistical modeling approach can be used in the design phase to help pick the most appropriate design, depending on factors such as the desired structural reliability, the available budget (cost factor) and other project restrictions. As explained before, different levels of failure can be defined for each structure, and in examining how and when the structures fail in these ways, it is possible to measure their reliability as a function of time. Different survival functions can be then obtained for a given structure, one for each structural failure type. By comparing the reliability of one bridge to another, one can determine whether a certain increase in structural robustness -either via redundancy or via reinforcement- is worthwhile according to the engineer's utility function. As can be deduced from Figure 3, the two possible bridges are the same length and height, but the second one (case B) has 3 more trusses connecting the top and bottom beam and is thus more structurally redundant. If the trusses have the same dimensions, the second bridge should have higher reliability than the first one (case A) for a longer period of time. Regardless the failure definition for the first bridge, the second bridge will need more time to suffer from a similar failure. Analogously, a bridge with reinforced components or improved individual reliability is expected to be more reliable.

Let us consider three different types of failure. Type 1 failure corresponds to slight damage, where the structure is no longer as robust as it was at the beginning but it can still be expected to perform the function it was built for. Type 2 failures correspond to severe damage, where the structure is no longer stable but it is still standing. Finally, Type 3 failure corresponds to complete structural failure, or col-lapse. Now we have four states to describe the structure, but only two (failed or not failed) to describe each

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component of the structure. We can track the state of the structure by tracking the states of its components. Also, we can compare the reliabilities of the two different structures over time, taking into account that different numbers of component failures will correspond to each type of structural failure depending on the structure. For example, a component failure in Case A could lead to a Type 2 failure (severe damage), while it would only lead to a Type 1 failure (slight damage) in the Case B bridge. In other words, for Case B it will take at least two components to fail in the same section of the bridge before the structure experiences a Type 2 failure.

The first step in order to develop a numerical example will be to define the logical topology for each design. For Case A, only one minimal path must be considered since the structure will be severely damaged (the kind of "failure" we are interested in) whenever one of its components fails. However, for Case B a total of 110 minimal paths were identified. The structure will not experience a type 2 failure if, and only if, all components in any of those minimal paths are still operative (Faulin et al., 2008).

As a next step, we will allocate reliability at the component level such that the system meets its overall target. We will assume the desired reliability after 15 years is 0.90. Finding alternative designs that meet the desired target would allow the stake holders to select the most attractive (cheapest) alternative that meets the given target. Once the optimum individual component reliability is obtained for the different designs, decisions can be made regarding the individual components.

For Case A, obtaining the individual components target reliabilities (and assuming no other feasibility constrains) is a straight forward matter. A reliability block diagram (RBD) can visually display the reliability logic for a system and is widely used in survival analysis. The RBD for Case A is show in Figure 4.

$1 \rightarrow 2 \rightarrow 3 \rightarrow 4$	\rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow	8 -> 9 -> 10 -	→ 11 → 12 → 13
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Figure 4: Reliability Block Diagram for Case A

Since in Case A only one minimal path is exists, all components are equally important and therefore the individual component reliability can be obtained as:

$$R_i(10 y ears) = R_{system}(10 y ears)^{\overline{N}}$$

Where R_i is the individual component target reliability, R_{system} is the system target reliability and N is the number of components. The reliability for each individual component can then be calculated as 0.9919.

For Case B however, the component optimum reliability cannot be obtained as easily because the criticality of the different components is not the same. The RBD for Case B is show in Figure 5.

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Figure 5: Reliability Block Diagram for Case B

From the RBD, the system reliability function which is function of its component reliability can be derived. Let's define the reliability importance of each component as follows (Wang et al. 2004):

$$I_{k}(t) = \frac{\partial R_{s}(t)}{\partial R_{k}(t)}$$

Figure 6 shows the reliability importance for the different components in Case B. Notice that components 9 and 5 are identified as the most critical components, that is, a change in the reliability of these components will have the most impact on the reliability of the system.



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In order to obtain optimum reliabilities for the components, we will use the reliability allocation method described by Mettas A. (2000). For the sake of simplicity, equal initial reliabilities, equal feasibilities and equal maximum reliabilities are assumed. The optimum component reliabilities are shown in Table 1.

Component	R(10 years)	Component	R(10 years)									
1	0.9536	9	0.9269									
2	0.9536	10	0.9536									
3	0.9536	11	0.9536									
4	0.9536	12	0.9536									
5	0.9269	13	0.9536									
6	0.9505	14	0.9536									
7	0.9505	15	0.9505									
8	0.9505	16	0.9536									

Table 1: Target Reliabilities for Components in Case B

Once the component target reliabilities are obtained, design choices regarding what components should be utilized can be made. Tables 2 and 3 contain failure-time distributions of components that would meet these requirements for designs A and B respectively. Figure 7 shows the survival (reliability) functions obtained in each case after a Type 2 failure–notice that similar curves could be obtained for other types of failures. This survival function shows the probability that each bridge will not have failed –according to the definition of a Type 2 failure– after some time (expressed in years). As expected, both Cases A and B meet the reliability requirement of 0.9 at 15 years. In this example, both cases are similar in reliability until the target time of 15 years after which Case B drops quicker in reliability. This is due to the fact that in order for design A to meet the reliability target, components that are much more reliable are needed, which might be a costly option. Notice that this conclusion holds only for the current values in Tables 2 and 3; should the shape and scale parameters change –e.g. by changing the quality of the components–, the survival functions would be different.

Component	Distribution	Shape	Scale	Component	Distribution	Shape	Scale
1	Weibull	4	50.0	8	Weibull	4	50.0
2	Weibull	4	50.0	9	Weibull	4	50.0
3	Weibull	4	50.0	10	Weibull	4	50.0
4	Weibull	4	50.0	11	Weibull	4	50.0
5	Weibull	4	50.0	12	Weibull	4	50.0
6	Weibull	4	50.0	13	Weibull	4	50.0
7	Weibull	4	50.0	-	-	-	-

Table 2: Failure-time distributions at component level for Case A

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Component	Distribution	Shape	Scale	Component	Distribution	Shape	Scale						
1	Weibull	4	28.6	9	Weibull	4	32.1						
2	Weibull	4	32.1	10	Weibull	4	31.6						
3	Weibull	4	28.6	11	Weibull	4	31.6						
4	Weibull	4	31.6	12	Weibull	4	31.6						
5	Weibull	4	32.1	13	Weibull	4	32.1						
6	Weibull	4	32.1	14	Weibull	4	32.1						
7	Weibull	4	32.1	15	Weibull	4	32.1						
8	Weibull	4	32.1	16	Weibull	4	32.1						

Table 3: Failure-time distributions at component level for Case B



Figure 7: Survival (Reliability) functions for alternative designs

Because both alternatives have been designed to meet the same reliability target, the decision can now be made based on economic factors such as the acquisition cost of components that will meet the desired reliability. Other metrics of interest such as availability and maintenance costs of the structure would have to include additional parameters such as repair distributions, cost of maintenance, cost and frequency of inspections, efficiency of repairs, etc. DES can then be used to expand the scope of the analysis.

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5. Experiment 2: Structural Availability

As shown in the previous example, analytical solutions may be used in a variety of situations. However, as more complex and realistic factors need to be taken into account, analytical approaches cannot provide answers without significant assumptions that may compromise the usefulness of the results. For example, as repair distributions, efficiencies of the repairs, cost and availability of resources, dependencies between components, preventive and condition based maintenances are taken into account, analytical approaches will need to make simplifying assumptions in order to obtain mathematical models. Nevertheless, DES does not have such limitations.

With DES, one can consider the effect of maintenance policies and track the structural availability of the system, as well as the associated costs of those repairs. For the purpose of illustrating our methodology, we will assume that in the previous example, Case B is found to be more economically feasible than case A and therefore is selected as the preferable design. We will also assume that historical data is available for the estimation of repair time distributions for the trusses (Table 4). We will select three plausible maintenance policy scenarios and estimate both the availability of the bridge over a mission time of 100 years as well as the maintenance costs. Case I will assume no preventive maintenance is performed, that is, repairs will be performed when a truss failure occurs. Case II will assume preventive maintenances with a frequency of 5 years and an efficiency of the repair of 70%. That is every 5 years, the structure will undergo preventive maintenance, effectively refurbishing the accumulated damage of each truss by 70%. Case III will assume a condition based maintenance with biyearly inspections. An inspection that finds a truss within the last 5% of its useful life will trigger a replacement of the truss. Table 4 shows preventive maintenance time distributions. For purposes of a cost analysis, we will assume a repair upon failure of a truss costs \$10,000, a preventive maintenance will cost \$1,000, the overall cost of inspecting the bridge is \$5,000 and a truss repair triggered by an inspection discovering a critically degraded condition costs \$5,000.

	Repair-ti	me		Preventive maintenance time						
Component	Distribution	Shape	Scale	Component	Distribution	Shape	Scale			
1	Weibull	2	0.5	1	Weibull	2	0.3			
2	Weibull	1.8	0.5	2	Weibull	1.8	0.3			
3	Weibull	1.8	0.3	3	Weibull	1.8	0.2			
4	Weibull	2	0.3	4	Weibull	2	0.2			
5	Weibull	2	0.5	5	Weibull	2	0.3			
6	Weibull	1.8	0.5	6	Weibull	1.8	0.3			

Table 4: Repair-time and preventive maintenance time distributions at component level

7	Weibull	1.8	0.3	7	Weibull	1.8	0.2
8	Weibull	1.8	0.3	8	Weibull	1.8	0.2
9	Weibull	2	0.5	9	Weibull	2	0.3
10	Weibull	1.8	0.5	10	Weibull	1.8	0.3
11	Weibull	1.8	0.3	11	Weibull	1.8	0.2
12	Weibull	1.8	0.3	12	Weibull	1.8	0.2
13	Weibull	2	0.5	13	Weibull	2	0.3
14	Weibull	1.8	0.5	14	Weibull	1.8	0.3
15	Weibull	1.8	0.5	15	Weibull	1.8	0.3
16	Weibull	1.8	0.5	16	Weibull	1.8	0.3

Using the proposed DES approach, the structural availability over time, specifically the probability that the structure under each scenario will be operative – not suffering a Type 2 or Type 3 failure – at any given time, can be obtained. Figure 8 shows the availability functions obtained for each alternative maintenance policy over a mission time of 100 years. Figure 9 and 10 show the expected costs and system failures over time for each alternative. Notice that the costs in this analysis do not include the cost of a system failure which is an important factor, particularly where safety is at risk. The expected costs and number of system failures can be used to factor in this additional cost. From expected costs and number of system failures it can be concluded that the interval preventive maintenance policy is most desirable in the first 40 years of the life of the bridge. As the bridge ages, a more aggressive policy such as regular inspections of the condition of the bridge seem to be more efficient. Note that these conclusions hold for the assumed inputs (e.g. failure and repair distributions, repair efficiencies, maintenance frequencies, etc.). Alternative inputs may lead to different conclusions.



Figure 8: Availability function for alternative maintenance policies

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Figure 9: Expected number of Type 2 or 3 failures for alternative maintenance policies



Figure 10: Expected cost for alternative maintenance policies

Figure 11 shows in more detail the simulated lifetime of the structure under the conditions in Case I, notice how the availability score A(t) fluctuates over time, depending on the time length since the previous maintenance. See Appendix for the table of values for Case II and III.

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	Point Results at Preselected System Times														
Time	A(t)	Cost(t)	Failures(t)	Time	A(t)	Cost(t)	Failures(t)	Time	A(t)	Cost(t)	Failures(t)	Time	A(t)	Cost(t)	Failures(t)
1	1	1	0	26	0.9889	59611	0.3343	51	0.9938	205475	1.4572	76	0.9934	347606	2.4776
2	1	4	0	27	0.986	66920	0.4009	52	0.9951	210396	1.4858	77	0.9932	352874	2.5117
3	1	21	0	28	0.9869	74725	0.474	53	0.9935	215588	1.519	78	0.994	357974	2.5441
4	1	52	0	29	0.9857	82488	0.5479	54	0.9926	220941	1.556	79	0.9943	363199	2.5758
5	1	115	0	30	0.9844	90359	0.6219	55	0.9923	226560	1.5977	80	0.9924	368478	2.6119
6	1	229	0	31	0.9876	98170	0.6929	56	0.9916	232192	1.6373	81	0.9929	373819	2.6467
7	1	412	0	32	0.985	105940	0.7698	57	0.9935	238035	1.6777	82	0.9946	379147	2.6794
8	1	673	0.0001	33	0.9855	113522	0.8404	58	0.9901	243883	1.7217	83	0.9917	384337	2.712
9	1	1063	0.0003	34	0.9889	120762	0.9021	59	0.9909	249795	1.7687	84	0.9928	389637	2.7507
10	1	1615	0.0008	35	0.9881	127770	0.969	60	0.9922	255914	1.8123	85	0.9936	395023	2.7863
11	0.9996	2371	0.0016	36	0.9898	134347	1.0221	61	0.9914	262245	1.862	86	0.9934	400422	2.8221
12	0.9999	3364	0.0023	37	0.9911	140618	1.069	62	0.9929	268442	1.9088	87	0.9923	405901	2.8624
13	0.9991	4715	0.0044	38	0.9928	146516	1.1152	63	0.9901	274533	1.9549	88	0.9925	411390	2.8998
14	0.999	6300	0.0083	39	0.9923	152039	1.1543	64	0.9926	280678	1.9988	89	0.9925	416928	2.9407
15	0.9993	8180	0.0129	40	0.9942	157120	1.1865	65	0.9917	286677	2.0455	90	0.9917	422444	2.9809
16	0.9986	10524	0.0195	41	0.9953	161918	1.2153	66	0.99	292627	2.0906	91	0.9923	428073	3.0262
17	0.999	13307	0.0274	42	0.9955	166543	1.2396	67	0.9916	298489	2.1323	92	0.9939	433788	3.0664
18	0.9975	16506	0.0409	43	0.9948	170761	1.2643	68	0.9939	304269	2.1744	93	0.9916	439535	3.1082
19	0.9957	20102	0.056	44	0.9955	175012	1.2867	69	0.9923	309931	2.2122	94	0.9923	445063	3.148
20	0.996	24184	0.0767	45	0.9958	179136	1.3081	70	0.992	315458	2.2534	95	0.9932	450742	3.1848
21	0.994	28946	0.1022	46	0.9962	183302	1.3288	71	0.9919	320996	2.2946	96	0.9935	456350	3.2242
22	0.9929	34082	0.1353	47	0.9973	187477	1.3496	72	0.9925	326469	2.3363	97	0.9925	461923	3.2653
23	0.9913	39765	0.175	48	0.9956	191759	1.3745	73	0.9936	331707	2.3685	98	0.993	467556	3.303
24	0.9919	45968	0.2231	49	0.9953	196180	1.4008	74	0.9927	337150	2.4067	99	0.9907	473256	3.3453
25	0.9899	52643	0.2757	50	0.9944	200769	1.4292	75	0.9938	342395	2.4446	100	0.9923	478717	3.3858

Figure 11: Resulting Values for Case I

Even though we have analyzed only the above three different maintenance policies using simulation, it is clear that DES is a useful tool to conduct a variety of analysis for real world scenarios. The results from similar analysis using simulation can then be used by decision makers to make the right choice in terms of safety, availability and financial considerations.

6. Conclusions

The advantages of using probabilistic methods to estimate reliability, availability and other metrics of interest in time-dependent building and civil engineering structures has been discussed. Among the available methods, discrete-event simulation (DES) seems to be the most realistic choice. DES offers clear advantages over other approaches, namely: (1) the opportunity of creating models which accurately reflect the structure's characteristics and behavior –including possible dependences among components' failure and repair times–, and (2) the possibility of obtaining additional information about the system's internal functioning and about its critical components. Therefore, a simulation-based approach is recommended for practical purposes, since

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it can consider details such as multi-state structures, dependencies among failure and repair-times, or non-perfect maintenance policies. The numerical examples discussed in this paper provide some insight on how DES can be used to estimate structural reliability and availability functions when analytical methods are not available, how it can contribute to detect critical components in a structure that should be reinforced or improved, and how to make better design decisions that consider not only the construction of such structures but also possible maintainability policies.

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Appendix

	Point Results at Preselected System Times														
Time	A(t)	Cost(t)	Failures(t)	Time	A(t)	Cost(t)	Failures(t)	Time	A(t)	Cost(t)	Failures(t)	Time	A(t)	Cost(t)	Failures(t)
1.001	1	0	0	26.026	0.9993	91844.7	0.0198	51.051	0.9948	258815.8	0.4741	76.076	0.9986	426276.4	0.916
2.002	1	5	0	27.027	0.9992	93338.7	0.023	52.052	0.9966	263169.8	0.4976	77.077	0.998	428847.4	0.9247
3.003	1	20	0	28.028	0.999	95210.7	0.0279	53.053	0.9945	267808.8	0.5279	78.078	0.9981	431701.4	0.936
4.004	1	47	0	29.029	0.9986	97488.7	0.0351	54.054	0.9944	272879.8	0.5587	79.079	0.9975	434824.4	0.9486
5.005	0	16094.5	0	30.03	0	115990.5	0.0435	55.055	0.0002	293555.5	0.59	80.08	0.0238	453833.2	0.9629
6.006	1	16108.5	0.0001	31.031	0.9985	117801.5	0.0518	56.056	0.9965	297154.5	0.6136	81.081	0.9985	456184.2	0.9742
7.007	1	16147.5	0.0001	32.032	0.9987	120163.5	0.0588	57.057	0.9958	301248.5	0.6361	82.082	0.9978	458951.2	0.9848
8.008	1	16221.5	0.0001	33.033	0.9992	122965.5	0.0677	58.058	0.9957	305530.5	0.657	83.083	0.9973	461992.2	0.9963
9.009	1	16339.5	0.0001	34.034	0.9981	126214.5	0.079	59.059	0.9961	310057.5	0.6791	84.084	0.9973	465285.2	1.0109
10.01	0	32529.1	0.0002	35.035	0	145603.8	0.0956	60.06	0.0005	330198.7	0.7035	85.085	0.0486	484428.3	1.0274
11.011	1	32620.1	0.0005	36.036	0.998	148082.8	0.1084	61.061	0.9972	333419.7	0.7244	86.086	0.9981	487020.3	1.0413
12.012	1	32770.1	0.0005	37.037	0.9977	151282.8	0.1212	62.062	0.9974	336965.7	0.7406	87.087	0.9977	490051.3	1.0522
13.013	1	33031.1	0.0006	38.038	0.9966	154905.8	0.1359	63.063	0.9973	340616.7	0.7558	88.088	0.9968	493296.3	1.0664
14.014	1	33436.1	0.0007	39.039	0.9958	158911.8	0.1581	64.064	0.997	344345.7	0.7715	89.089	0.9964	497008.3	1.0838
15.015	0	49889.8	0.0011	40.04	0	179134.5	0.183	65.065	0.0022	363768.9	0.7875	90.09	0.0819	516478.9	1.1024
16.016	1	50185.8	0.0015	41.041	0.9962	182427.5	0.2041	66.066	0.9977	366382.9	0.8006	91.091	0.9972	519435.9	1.1201
17.017	1	50631.8	0.0017	42.042	0.9967	186303.5	0.2221	67.067	0.9979	369243.9	0.8109	92.092	0.9967	522624.9	1.1343
18.018	0.9998	51232.8	0.0022	43.043	0.994	190677.5	0.2445	68.068	0.9969	372327.9	0.8241	93.093	0.9959	526130.9	1.1489
19.019	0.9998	52029.8	0.003	44.044	0.9952	195547.5	0.271	69.069	0.9973	375466.9	0.8383	94.094	0.9948	530126.9	1.1695
20.02	0	69011.6	0.0041	45.045	0	216254.5	0.3024	70.07	0.0048	394518.9	0.8503	95.095	0.1206	549774.6	1.1889
21.021	0.9996	69657.6	0.0067	46.046	0.9965	219882.5	0.3261	71.071	0.9988	396873.9	0.8635	96.096	0.9969	552949.6	1.2096
22.022	0.9995	70542.6	0.0082	47.047	0.9947	224285.5	0.3496	72.072	0.9978	399485.9	0.8744	97.097	0.9972	556441.6	1.2256
23.023	0.9999	71748.6	0.0097	48.048	0.9947	229076.5	0.3774	73.073	0.9981	402249.9	0.8842	98.098	0.9952	560231.6	1.2459
24.024	0.9992	73161.6	0.0129	49.049	0.9918	234178.5	0.413	74.074	0.9978	405205.9	0.8953	99.099	0.9957	564241.6	1.2642
25.025	0	90783.7	0.0157	50.05	0.0001	255029.8	0.4431	75.075	0.0136	424082.4	0.9064	100.1	0.1593	583962.2	1.2874

Figure 12: Resulting Values for Case II

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						Point Res	ults at Pres	elected	System	Times					
Time	A(t)	Cost(t)	Failures(t)	Time	A(t)	Cost(t)	Failures(t)	Time	A(t)	Cost(t)	Failures(t)	Time	A(t)	Cost(t)	Failures(t)
1.001	1	0.1	0	26.026	0.41473	111840.678	0.12317	51.051	0.99994	256775.389	0.25287	76.076	0.74587	415098.924	0.39984
2.002	0.99999	4802.899	0	27.027	1	111897.478	0.12318	52.052	0.65737	268407.007	0.26648	77.077	0.99996	415360.824	0.39995
3.003	1	4812.599	0	28.028	0.37634	126985.011	0.15137	53.053	0.99996	268623.207	0.26653	78.078	0.75467	426650.03	0.40916
4.004	0.99914	9643.903	0	29.029	1	127064.111	0.15139	54.054	0.63688	280791.545	0.28129	79.079	0.99988	426912.93	0.4093
5.005	1	9696.503	0	30.03	0.35857	141969.015	0.17445	55.055	0.99999	280984.045	0.28135	80.08	0.76582	438253.278	0.41896
6.006	0.99529	14625.265	0.00002	31.031	1	142078.615	0.1745	56.056	0.62465	293473.428	0.29607	81.081	0.99992	438510.578	0.41911
7.007	0.99998	14759.465	0.00008	32.032	0.3626	156346.734	0.19118	57.057	0.99996	293668.828	0.29613	82.082	0.77064	450011.973	0.42941
8.008	0.98529	19913.687	0.00013	33.033	0.99998	156480.334	0.19126	58.058	0.61767	306305.094	0.31028	83.083	0.9999	450263.473	0.42953
9.009	0.99996	20169.487	0.0002	34.034	0.38899	169782.226	0.2018	59.059	0.99998	306473.694	0.31037	84.084	0.7715	461913.257	0.44072
10.01	0.96513	25703.262	0.00071	35.035	0.99997	169948.726	0.20189	60.06	0.61787	319167.762	0.32367	85.085	0.99997	462158.357	0.4408
11.011	0.99982	26093.262	0.001	36.036	0.43556	182113.967	0.20766	61.061	0.99997	319341.462	0.32371	86.086	0.77543	473921.827	0.45249
12.012	0.92967	32241.186	0.00247	37.037	0.99995	182313.567	0.20774	62.062	0.62383	331901.963	0.33641	87.087	0.99989	474160.127	0.45266
13.013	0.99977	32745.886	0.00288	38.038	0.49218	193387.939	0.21099	63.063	0.99993	332096.063	0.33654	88.088	0.78068	486016.924	0.46396
14.014	0.87516	39756.65	0.00581	39.039	0.99995	193614.739	0.21107	64.064	0.63711	344444.244	0.34709	89.089	0.99993	486252.024	0.4641
15.015	0.99965	40333.65	0.00626	40.04	0.55707	203857.395	0.21381	65.065	0.99991	344647.044	0.34721	90.09	0.78461	498156.957	0.47541
16.016	0.80691	48500.645	0.01287	41.041	0.99994	204116.395	0.21389	66.066	0.65393	356708.938	0.35716	91.091	0.99992	498403.657	0.47551
17.017	0.99968	48998.445	0.01324	42.042	0.62792	213974.679	0.21746	67.067	0.99995	356931.838	0.35726	92.092	0.78456	510367.861	0.48642
18.018	0.72601	58542.485	0.02384	43.043	0.99993	214242.379	0.2176	68.068	0.67318	368740.27	0.36601	93.093	0.99995	510609.861	0.48652
19.019	0.99979	58828.085	0.02405	44.044	0.67615	224049.866	0.22326	69.069	0.99997	368971.57	0.3661	94.094	0.79307	522526.504	0.49738
20.02	0.63303	70080.917	0.04171	45.045	0.99994	224327.166	0.2234	70.07	0.6932	380501.244	0.37403	95.095	0.99994	522770.804	0.4975
21.021	1	70094.517	0.04171	46.046	0.6996	234382.461	0.23096	71.071	0.99997	380743.744	0.37419	96.096	0.80421	534639.285	0.50786
22.022	0.54531	82987.877	0.06536	47.047	0.99989	234650.361	0.23111	72.072	0.71406	392095.867	0.38251	97.097	0.99994	534885.885	0.50799
23.023	0.99999	83013.077	0.06537	48.048	0.69355	245215.35	0.24129	73.073	0.99992	392345.867	0.38267	98.098	0.81085	546662.383	0.51806
24.024	0.47219	97018.209	0.0936	49.049	0.99992	245472.85	0.24143	74.074	0.73389	403576.938	0.39095	99.099	0.99994	546916.183	0.51814
25.025	1	97059.009	0.09365	50.05	0.68028	256539.189	0.25276	75.075	0.99989	403843.638	0.39114	100.1	0.82007	558656.756	0.52839

Figure 13: Resulting Values for Case III

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