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An iterative biased-randomized heuristic for the fleet size and mix vehicle-routing problem with backhauls

Javier Belloso[a], Angel A. Juan[b] and Javier Faulin[a]

[a] *Department of Statistics and Operations Research, Public University of Navarre, 31006, Pamplona, Spain*
[b] *Department of Computer Science, Multimedia, and Telecommunication – IN3, Open University of Catalonia, 08018, Barcelona, Spain*
E-mail: javier.belloso@unavarra.es [Belloso]; ajuanp@uoc.edu [Juan]; javier.faulin@unavarra.es [Faulin]

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Abstract

This paper analyzes the fleet mixed vehicle-routing problem with backhauls, a rich and realistic variant of the popular vehicle-routing problem in which both delivery and pick-up customers are served from a central depot using a heterogeneous and configurable fleet of vehicles. After a literature review on the issue and a detailed description of the problem, a solution based on a multistart biased-randomized heuristic is proposed. Our algorithm uses an iterative method that relies on solving a series of smaller instances of the homogeneous-fleet version of the problem and then using these subsolutions as partial solutions for the original heterogeneous instance. In order to better guide the exploration of the solutions space, the algorithm employs several biased-randomized processes: a first one for selecting a vehicle type; a second one for sorting the savings list; and a third one to define the number of routes that must be selected from the homogenous-fleet subsolution. The computational experiments show that our approach is competitive and able to provide 20 new best-known solutions for a 36-instance benchmark recently proposed in the literature.

Keywords: vehicle-routing problem with backhauls; heuristics; biased randomization; multistart algorithms; fleet size and mix vehicle-routing problem

1. Introduction

Road transportation is the predominant way of transporting goods in many world regions. This explains the relevance of rich and real-life vehicle-routing problems (VRPs), since efficient route planning can significantly reduce transportation costs and offer a better service to customers (Caceres et al., 2014). This paper analyzes the fleet mixed vehicle-routing problem with backhauls (FSMVRPB), in which both delivery and pick-up customers are served from a central depot using a heterogeneous and configurable fleet of vehicles. This rich and realistic variant combines the aspects

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of two well-known VRP versions, that is, the fleet size and mix VRP (FSMVRP), initially described in Golden et al. (1984), and the VRP with backhauls (VRPB), introduced in Golden et al. (1985).

The FSMVRP relates to finding the best fleet composition considering that vehicle costs can be divided into fixed and variable ones. Fixed costs are directly allocated when a vehicle of certain type is assigned to a route—despite the characteristics of the assigned route—whereas variable costs might depend upon the type of vehicle employed and the route length. The objective here is the minimization of the total costs required to deliver the ordered products to the customers. Having to deal with heterogeneous fleets is quite usual in most practical situations. In fact, when considering fixed and variable costs, even two vehicles with the same capacity can be considered as different—in terms of costs—due to factors such as depreciation, insurance, maintenance, or operating costs (Hoff et al., 2010). In the VRPB, customers are also allowed to return goods to the depot, although it is assumed that each specific customer cannot receive and return items simultaneously—if that were the case, it would be considered as two different customers during the problem-modeling process.

In a recent work, Salhi et al. (2013) introduced the combined problem, motivated its importance, and proposed a solving approach for the FSMVRPB. These authors also proposed a new set of 36 instances for benchmarking purposes. Following their seminal work, this paper offers a review of related work and proposes a multistart biased-randomized heuristic to solve the FSMVRPB. Our algorithm uses an iterative method that relies on solving a series of smaller instances of the homogeneous-fleet version of the problem and then using these subsolutions as partial solutions for the original heterogeneous instance. Also, the algorithm employs several biased-randomized processes in order to better guide the search process.

The remainder of the paper is organized as follows. Section 2 reviews the related works. Section 3 provides a description of the FSMVRPB. Section 4 explains the main ideas of our algorithm. Section 5 describes the pseudo-code details—which might be especially useful for implementation purposes. Section 6 contains the computational experiments carried out in order to test the efficiency of our approach. Finally, Section 7 summarizes the main contributions of our work.

2. Literature review

The FSMVRPB considering both fixed and variable costs was formally introduced by Salhi et al. (2013). Before that, Tütüncü (2010) analyzed a simplified version of the problem including heterogeneous fleet as well as fixed and variable costs, but without considering the fleet size and mix component—that is, the fleet size and composition was given in advance. Our work follows the one by Salhi et al. (2013) and since no other previous work in the literature has examined the FSMVRPB, we will review separately the two components of the problem: the VRPB and FSMVRP.

2.1. The VRP with backhauls

The most common and intuitive approach described in the literature to solve this problem has the following scheme: (1) solving the linehaul problem; (2) solving the backhaul problem; and (3) combining both partial solutions into a global one. The first two subproblems are, in fact, the same problem, whereas the third one is exclusive of the backhauling variant. One of the first methods proposed to solve the VRPB was designed by Deif and Bodin (1984). These authors follow the aforementioned sequential approach and explore the solutions space by adapting the popular savings heuristic (CWS) developed by Clarke and Wright (1964). The savings list used by the CWS heuristic is adjusted to the VRPB case. This is done by incorporating the so-called precedence constraint, that is, backhaul customers cannot be visited until all deliveries have been completed. The criteria for the selection of the interface edge (the one connecting the linehaul route with the backhaul route) is critical in this scenario, since it can tremendously improve the quality of each solution.

A similar approach is presented in Golden et al. (1985), who proposed a heuristic procedure based on the insertion principle: routes are initially generated for linehaul customers; then, backhaul customers are inserted into routes according to an insertion criterion. They use a penalty factor to impose the precedence constraint. This penalization forces the backhaul route to be introduced at the end of the linehaul route. Goetschalckx and Jacobs-Blecha (1989) proposed an algorithm that makes use of a mapping method to introduce the precedence constraint and generate two required clusters, one for linehaul customers and another one for backhaul customers. The two sequences are managed separately to create feasible routes, and then routes are merged. Another approach, proposed by Goetschalckx and Jacobs-Blecha (1993), is classified as a "cluster-first-and-route-second" algorithm. The cluster creation is managed as a generalized assignment problem and the route creation as a travelling salesman problem with the constraint that only one interface from delivery to pick-up points is permitted in each route. As explained before, the best sequence of customers' visits in the route is highly dependent on the selection of the link interface. Using the same cluster-first-and-route-second logic, Anily (1996) introduced another algorithm in which the clustering phase is accomplished by a modified circular partitioning heuristic. Once the customers are assigned to clusters, the process continues with the construction of travelling salesman tours through all the clusters (either linehaul or backhaul but not both). Finally, linehaul clusters are linked to backhaul clusters, and a route generation phase is initiated in order to determine the optimal connections between the depot and the clusters. The popular sweep algorithm from Gillett and Miller (1974) can be classified into this type of cluster-first-and-route-second approaches. Also, Toth and Vigo (1999) proposed another heuristic based on a K-tree Lagrangian relaxation formulated by Fisher (1994) to the VRP. They introduce the idea of asymmetric costs between two customers, and make use of lower bounds produced by a Lagrangian approach.

Some other methods designed for the general VRP can easily include the precedence constraint. This is the case of the improvement/exchange heuristics, which add the restriction that linehaul and backhaul customers should not be mixed during the process. In this direction, we can consider 2-opt and 3-opt procedures proposed by Lin (1965), which can be easily applied separately to the sets of linehauls and backhauls within each route.

VRPB literature since 2000 is extensively focused on metaheuristic methods. In this context, Osman and Wassan (2002) developed a tabu search (TS) algorithm. They solve the problem by using a metaheuristic based on savings, insertion, and assignment approaches. Brandão (2006) proposed a similar solution with his TS algorithm. Wassan (2007) designed a TS enhanced by adaptive memory programming. Ropke and Pisinger (2006) analyzed the VRPB as an extension of the rich pick-up and delivery problem with time windows. They presented a large neighborhood search heuristic to solve it. A detailed new classification of the problem was introduced by Parragh et al. (2008). Different backhauling strategies using ants are compared in Reimann and Ulrich (2006). Wassan et al. (2009) and Gajpal and Abad (2009) proposed an ant colony system to solve the VRPB. Tütüncü et al. (2009) proposed a decision support system based on a greedy randomized adaptive search with memory programming (GRASP). Zachariadis and Kiranoudis (2012) proposed a metaheuristic local search approach where different customer sequences were exchanged. Cuervo et al. (2014) proposed an iterated local search algorithm that produces high-quality results.

In recent years, environmental issues have attracted the attention of many researches due to the potential reduction of gas emissions and therefore costs. This result can be achieved by combining pickups and deliveries in the same route to avoid empty vehicles during the return-to-depot stage. This characteristic has been analyzed in papers such as Berbeglia et al. (2010), Ubeda et al. (2011), and Sheu and Talley (2011). Similarly, our literature review on the VRPB can be complemented with the extensive review completed by Toth and Vigo (2014).

2.2. *Fleet size and mix vehicle-routing problem (FSMVRP)*

In the FSMVRP, all customers receive materials from the depot. The problem has several variants depending on the assumptions made on the fleet size (either limited or unlimited) and the consideration or not of both fixed and variable costs.

The FSMVRP was introduced by Golden et al. (1984) and considers variable costs. These costs are computed as the distance traveled. In this case, a limited fleet size is considered. These authors use a constructive heuristic based on the aforementioned saving heuristic. Using a similar cost structure and fleet configuration, several authors have used TS algorithms for solving the FSMVRP (Gendreau, 1999; Osman and Wassan 2002). Taillard (1999) applies the column generation method to obtain homogenous routes for all kind of vehicles. Then, he solves the set partitioning problem to obtain the final solution by ensuring that each node is visited exactly once. Renaud and Boctor (2002) apply a similar algorithm, but with some improvements.

Choi and Tcha (2007) use a column generation method to solve the problem. Liu and Shen (1999) apply a genetic algorithm and Subramanian and Penna (2012) implement a hybrid algorithm by combining an iterated local search and a set partitioning mechanism. Some authors also consider variable costs, as in Salhi et al. (2013). Taillard (1999) adopts a similar approach. The goal for the version of the problem when the fleet size is limited is to obtain a suitable utilization of the existing fleet. Taillard (1999) adapts his algorithm to the limited fleet version. Tarantilis et al. (2003) obtain and initial solution with a constructive heuristic, achieving improvements through random movements that are evaluated following different criteria. They use a simulated annealing methodology. Finally, excellent reviews on this problem can be found in Baldacci et al. (2008) and Hoff et al. (2010).

3. Problem description

The FSMVRPB model considered in this article is an extension of the model proposed in Salhi et al. (2013), being the main difference that in our model not only fixed but also variable costs associated to each vehicle can be taken into account. Our model also extends the model of Lee et al. (2008) by incorporating the presence of backhauls. Consider a graph $G = (V, A)$, where $V = \{v_0, v_1, \ldots, v_L, v_{L+1}, \ldots, v_n\}$ is a set of nodes including the depot, v_0, L delivery customers, and $B = n$ - L pick-up customers; and A is a set of arcs connecting those nodes. Each arc $(i, j) \in A$ is associated with a travel distance d_{ij} that is known in advance. Located at the depot, there is a heterogeneous fleet of vehicles of K different types. The capacity of each vehicle υ is denoted by Q_υ. Also, each vehicle υ has both fixed costs, f_υ, as well as variable costs, c_υ. Each customer i has an associated demand d_i. The goal of the FSMVRPB is to design a set of routes minimizing total costs (fixed plus variable ones) of serving the customers (both linehauls and backhauls) subject to the following constraints: (1) each route is assigned to exactly one vehicle; (2) each route departs from the depot, serves its assigned customers, and returns to the depot; (3) each customer is visited once by exactly one vehicle; and (4) the maximum load carried by each vehicle during a route cannot exceed the vehicle capacity.

4. Overview of our solution methodology

This section offers an overview of the approach we have developed to solve the FSMVRPB, which integrates some methods and techniques developed in our previous works (for different VRP variants) and then adds a final combination stage in which different routes are mixed to build complete solutions. As depicted in Fig. 1, our approach successively solves homogeneous versions of the problem—each of them using a different type of vehicle—in order to solve the heterogeneous version of the problem. Then, it combines parts of the homogeneous solutions to generate a solution for the heterogeneous case. This methodology builds up on the "successive approximations method" (SAM) originally proposed by Juan et al. (2014) to solve the heterogeneous VRP. SAM is a multiround process. At each round, a new subset of nodes and a new type of vehicle are selected following some specific criteria. Then, assuming an unlimited fleet of vehicles of this type, the associated homogeneous-fleet VRP is solved. After several rounds, a global solution for the heterogeneous VRP is obtained by merging routes from different homogeneous VRP solutions. Notice that the SAM approach transforms the challenge of solving a complex heterogeneous VRP into the challenge of solving a series of related homogeneous VRPs, each of them smaller than the original heterogeneous VRP. The process shown in Fig. 1 starts by randomly selecting the type of vehicle (each type of vehicle has a different capacity) to be used in the homogeneous version of the problem. Notice that only vehicles with a capacity larger than the maximum demand to be served can be considered at this stage. Then, the problem becomes homogeneous and it is solved assuming an unlimited number of vehicles of this type (CVRPB). In order to solve the homogeneous version of the problem we have used the algorithm presented in Belloso et al. (2015), which is based on a variant of the aforementioned CWS heuristic. Thus, the adapted heuristic makes use of a savings list that is manipulated in two steps. In the first step, the interface links (edges connecting one linehaul customer with one backhaul customer) are penalized using the approach proposed in Deif and Bodin (1984). This penalization makes them to be chosen at a later stage than if the traditional CWS heuristic was employed instead. Therefore, the precedence constraint (linehauls first, then backhauls) is satisfied.

In the second step, the savings list obtained in the CWS heuristic is randomized using a geometric probability distribution (Juan et al., 2011, 2015b; Dominguez et al., 2014). There are different approaches for randomizing a heuristic with the goal of better exploring the solutions space (Hemmati

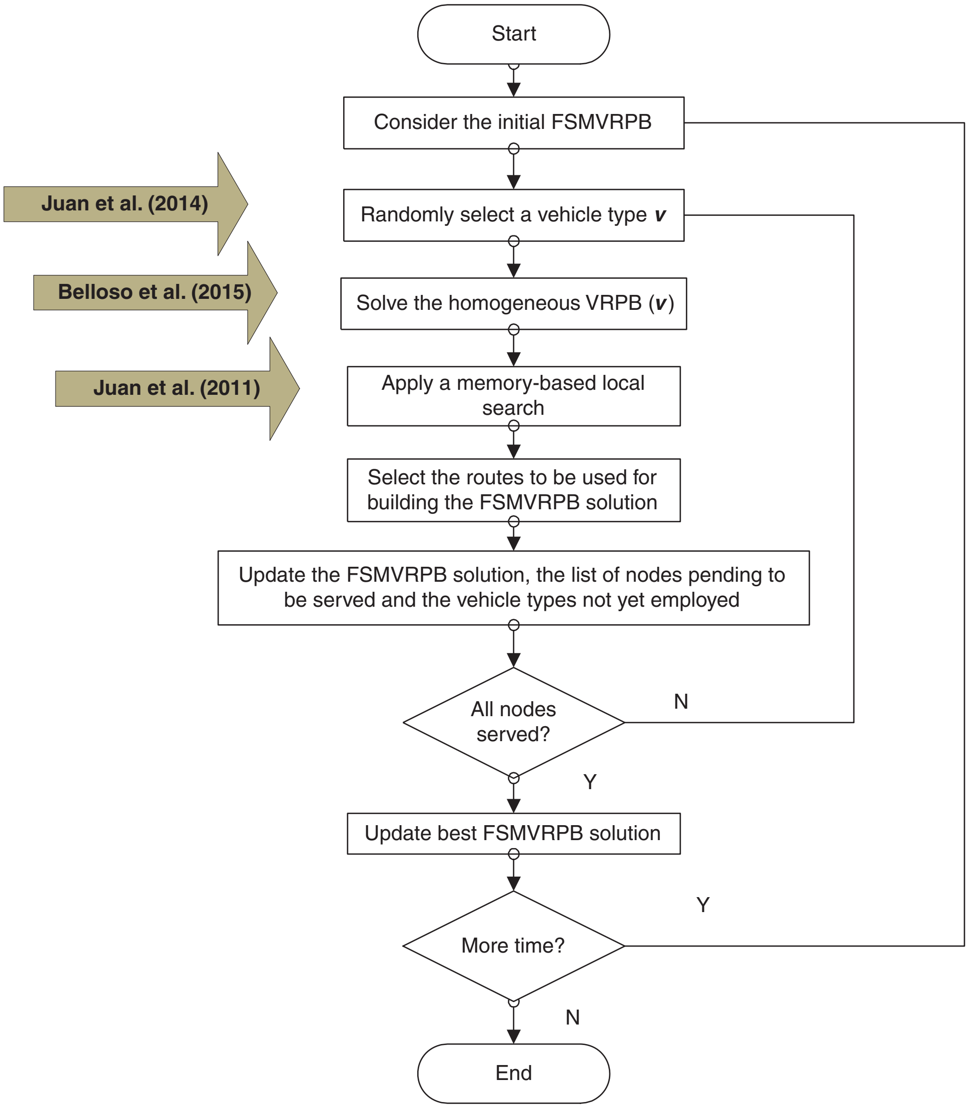

Fig. 1. Flowchart for the proposed methodology.

and Hvattum, 2017; Santos et al., 2016). One of these is the use of GRASP (Resende and Ribeiro, 2010). GRASP is a multistart process that randomizes a given heuristic to better explore the solution space. The solution is obtained by following a constructive sequence. At each step, a restricted list of candidates is ordered following a greedy function that evaluates the benefit of selecting each element. Then, a candidate is randomly selected for that restricted list, thus allowing the algorithm to generate different solutions at each iteration of the multistart process. In a classical GRASP, randomness is typically provided by a symmetric uniform distribution. However, in our approach we use the geometric probability distribution (a skewed one) to induce a biased-randomized selection process. That way, we add some degree of randomness into the CWS heuristic without destroying the logic behind it. This procedure improves and extends the initial ideas proposed in Faulín and Juan (2008). In other words, this biased-randomization technique ensures that edges with higher savings will have also a higher probability of being selected at each iteration of the solution-construction process (Juan et al., 2015b; Quintero-Araujo et al. 2017). Other variants of biased-randomization techniques have been also applied in the improvement of genetic algorithms. Some examples of this can be found in Reis et al. (2011), Morán-Mirabal et al. (2014), Gonçalves and Resende (2014), Brandão et al. (2015), and Gonçalves et al. (2016).

At this stage, the algorithm provides a solution for the homogeneous version of the problem. This solution is then improved throughout a memory-based local search process in which any route in the current solution is checked against a cache memory (hash map data structure in programming terminology) storing the best-known way of routing the set of nodes in the given route. If the same set of nodes is found in memory (hash map) and the cost is improved, the route included into the current solution is updated using the improved equivalent route stored in memory. Otherwise, the memory is updated with the route in the current solution. More technical details of this technique can be found in Juan et al. (2011). Once this local search process is finished, the next step is to randomly determine which routes in the homogeneous solution will be used as "building blocks" for the heterogeneous solution. The selected routes determine a set of customers already served and type of vehicles already used. Thus, the same procedure is iteratively run—using a different type of vehicle for solving the homogeneous version of the problem—until all customers have been served. At the end of this iterative approach, a complete solution for the heterogeneous version is obtained by aggregating all the routes selected from the solutions to the different homogeneous problems.

5. Algorithm pseudo-code

The methodology presented in the previous section is implemented through the "iterative homogeneous problems resolution operator," which is called from a multistart procedure each time a homogeneous VRPB needs to be solved. This multistart procedure ends when a time-based termination criterion is reached. Each new solution generated by this procedure for a given VRPB is compared with the best one obtained so far for the same problem. Figure 2 shows the algorithm pseudo-code. Lines 1, 2, and 3 represent the initialization of a particular homogeneous problem. Specifically, line 3 defines the homogeneous problem once the vehicle type is chosen in line 1. The customers to be visited are defined as inputs of the procedure. Line 4 calls to the heuristic that generates a solution for the homogeneous problem. Line 5 calls to the memory-based local search that tries to improve the current solution. At this point, part of the homogeneous solution is ready to be promoted to the heterogeneous solution. Lines 6 to 10 manage the selection of routes and update the list of available types of vehicles. If the current vehicle type is the last one in the list of vehicles, all routes in the homogeneous solution will be transferred to the final solution. Otherwise, a random number between 1 and the number of routes is selected. Line 11 summarizes the process of promoting routes to the final solution, while lines 12 and 13 represent the updated value of customers and vehicles. Line 14 checks if the termination criterion of the procedure is met. If this is the case, the iterative procedure returns the current solution; otherwise (Line 16) the procedure is invoked again with the nonserved customers and nonused vehicle types as input parameters.

```
procedure IHPO(customers, vehicles) // Iterative Homogeneous Problems Operator
// Iteratively solves homogeneous problems until all customers are served
  newVehType <- selectRandom(vehicles) // Select new type of vehicle randomly
  vehCap <- getCapacity(newVehType)
  newHomVRPB <- Initialization(customers, vehCap)
// Solve homogeneous problem using an efficient algorithm
  homSol <- solveHoVRPB(newHomVRPB)
// Use a cache structure to recall previously computed subroutes
  homoSol <- ImproveNodesOrder(homoSol)
// Select number of routes (numroutes) to be transferred to heterogeneous solution
  if {pending vehicles is 0} then
    numroutes <- routes(homoSol)
  else
    numroutes <- selectRandom(1…routes(homoSol))
  end if
  heteroSol <- addRoutes(homoSol(numroutes))
  pendingCustomers <- extractCustomers(homoSol(routes(homoSol)-numroutes)
  pendingVehicles <- extractVehicles(newVehType)
// Return final solution
  if {number of pendingVehicles is zero} then
    return heteroSol
  else
    return heteroSol & IHPO(pendingCustomers, pendingVehicles)
end procedure
```

Fig. 2. Pseudo-code for the main algorithm.

6. Experimental results

The FSMVRPB instances we consider in this paper are the ones recently proposed by Salhi et al. (2013), which are generated using the mixed fleet dataset of Golden et al. (1984) and the VRPB dataset of Toth and Vigo (1997). From the mixed fleet dataset, these instances take the mix fleet attributes (distances, fixed and variable costs, and vehicle capacities), while they take the backhaul percentages from the VRPB dataset. It should be noted that the original source of this FSMVRPB dataset is the dataset initially proposed by Christofides and Eilon (1969). Salhi et al. (2013) generated a set of 36 FSMVRPB instances using the 12 test problems of Golden et al. (1984) ranging from 20 to 100 customers. For each instance, following the conventions used in Toth and Vigo (1997), three new instances were generated using linehaul/backhaul (LH/BH) percentages of 50/50, 67/33, and 80/20. This was done by taking, as a backhaul customer, every first customer in series of two, three, and five customers, respectively. The algorithm described in this paper has been implemented as a Java application. We have solved the set of 36 instances in order to test the effectiveness of the proposed method.

A standard personal computer, Intel R Core TM i7 CPU M 640 @ 2.80 GHz, 3.42 GB RAM, and an MS XP Professional operating system was used to perform these tests. The results are summarized in Table 1, which contains the following information for each instance: name of the instance, number of customers, number of linehaul customers, number of backhaul customers, best-known solution (BKS) as reported by Salhi et al. (2013), our best solution (OBS) after 10 runs of the algorithm, the percentage gap between both values (computed as (OBS-BKS)/BKS), computing time employed in obtaining OBS, and optimal solution or lower bound as reported in Salhi et al.

Table 1
Computational results for the FSMVRPB

Problem	Size	Line	Back	BKS	OBS	Gap (%)	Time (seconds)	Optimal[a] or lower bounds
HWS1	20	10	10	**726.48**	734.03	1.04	10	720.7[a]
HWS2	20	13	7	**818.12**	820.99	0.35	15	818.12[a]
HWS3	20	16	4	848.59	**848.32**	−0.03	1	848.23[a]
HWS4	20	10	10	4350.65	**4342.48**	−0.19	2	4342.48[a]
HWS5	20	13	7	5366.39	**5357.98**	−0.16	8	5357.98[a]
HWS6	20	16	4	5875.23	**5872.52**	−0.05	6	5421.63[a]
HWS7	20	10	10	767.93	**729.50**	−5.00	15	729.50[a]
HWS8	20	13	7	872.97	**838.11**	−3.99	16	838.11[a]
HWS9	20	16	4	903.18	**890.76**	−1.38	18	890.76[a]
HWS10	20	10	10	4365.44	**4349.13**	−0.37	2	4349.12[a]
HWS11	20	13	7	5414.50	**5363.58**	−0.94	21	5363.58[a]
HWS12	20	16	4	5928.78	**5523.50**	−6.84	102	5465.40
HWS13	50	25	25	**1625.70**	1632.60	0.42	23	1480.75
HWS14	50	33	17	1811.63	**1793.26**	−1.01	14	1705.44
HWS15	50	40	10	**2018.93**	2030.37	0.57	65	1937.24
HWS16	50	25	25	**5561.67**	5562.94	0.02	16	5537.24
HWS17	50	33	17	**6570.39**	6571.77	0.02	4	6547.93[a]
HWS18	50	40	10	7599.08	**7413.27**	−2.45	19	7026.15
HWS19	50	25	25	1704.41	**1659.86**	−2.61	71	1509.41
HWS20	50	33	17	**2037.23**	2038.22	0.05	120	1926.03
HWS21	50	40	10	**2340.09**	2341.17	0.05	97	2196.32
HWS22	50	25	25	1774.71	**1746.28**	−1.60	11	1697.38
HWS23	50	33	17	2166.52	**2164.65**	−0.09	110	2028.35
HWS24	50	40	10	2430.88	**2425.83**	−0.21	23	2335.56
HWS25	75	37	38	1332.02	**1313.59**	−1.38	246	1228.48
HWS26	75	50	25	1421.04	**1412.69**	−0.59	95	1341.97
HWS27	75	60	15	**1534.65**	1543.10	0.55	145	1466.24
HWS28	75	37	38	**1617.85**	1631.70	0.86	65	1486.89
HWS29	75	50	25	**1799.76**	1808.46	0.48	45	1660.15
HWS30	75	60	15	**1990.46**	2001.79	0.57	197	1853.72
HWS31	100	50	50	**4943.29**	4990.84	0.96	310	4870.54
HWS32	100	66	34	6035.96	**6019.82**	−0.27	126	5901.57
HWS33	100	80	20	7601.29	**7593.25**	−0.11	43	6886.44
HWS34	100	50	50	**2465.41**	2516.30	2.06	241	2369.80
HWS35	100	66	34	**2971.98**	2998.21	0.88	156	2730.67
HWS36	100	80	20	**3533.90**	3563.17	0.83	253	3139.87
Average						−0.54	75.31	

(2013). Values in bold represent the BKS after our work. Notice that our algorithm is able to provide 20 new BKSs and offers an average negative gap of –0.54% with respect to the previous BKS values. Figure 3 also contributes to illustrate the efficiency of our approach by showing a visual comparison among the BKS, OBS, and the lower bound values for each instance. The results provided by our algorithm are particularly noteworthy taking into consideration the low computation times employed.

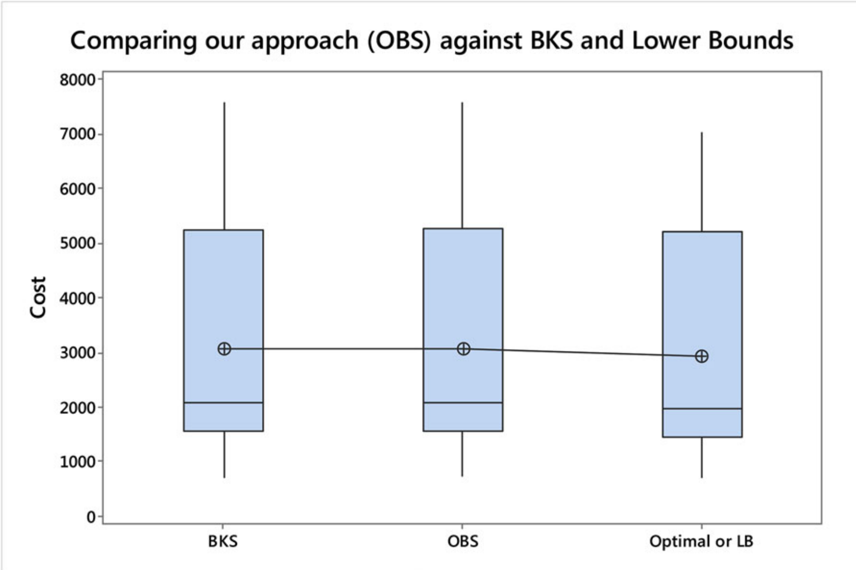

Fig. 3. Comparison of our approach with BKS and optimal or lower bound.

7. Conclusions

This paper presents an original and efficient metaheuristic approach to solve the FSMVRPB, a realistic variant of the VRP that has rarely been considered in the literature due to its inherent complexity. Our approach integrates different strategies that contribute to effectively deal with the problem, including the following: a successive approximation method that reduces the FSMVRPB to iteratively solving a series of homogeneous VRPB, a biased-randomized metaheuristic algorithm for solving the VRPB, and a memory-based local search. Despite its conceptual simplicity, the proposed approach is able to provide noticeable results when compared with other previous works, and it attains 20 new BKSs for the 36-instance set considered, also showing negative average gap and low computing times. As a future research line, we plan to incorporate even more realistic conditions to the problem by considering stochastic demands for the customers, which we plan to address by transforming the current metaheuristic approach into a simheuristic one as described in Juan et al. (2015a).

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References

Anily, S., 1996. The vehicle-routing problem with delivery and back-haul options. *Naval Research Logistics* 43, 3, 415–434.

Baldacci, R., Battarra, M., Vigo, D., 2008. Routing a heterogeneous fleet of vehicles. In Golden, B.L., Raghavan, S., Wasil, E.A. (eds). *The Vehicle Routing Problem: Latest Advances and New Challenges.* Springer, New York, pp. 3–27.

Belloso, J., Juan, A., Faulin, J., Serrano, A., 2015. Using multi-start biased randomization of heuristics to solve the vehicle routing problem with clustered backhauls. *Lecture Notes in Management Science* 7, 15–20.

Berbeglia, G., Cordeau, J.F., Laporte, G., 2010. Dynamic pickup and delivery problems. *European Journal of Operational Research* 202, 1, 8–15.

Brandão, J., 2006. A new tabu search algorithm for the vehicle routing problem with backhauls. *European Journal of Operational Research* 173, 2, 540–555.

Brandão, J.S., Noronha, T.F., Resende, M.G.C., Ribeiro, C.C., 2015. A biased random-key genetic algorithm for single-round divisible load scheduling. *International Transactions in Operational Research* 22, 5, 823–839.

Caceres, J., Arias, P., Guimarans, D., Riera, D., Juan, A., 2014. Rich vehicle routing problem: a survey. *ACM Computing Surveys* 47, 2, art. 32.

Choi, E., Tcha, D.W., 2007. A column generation approach to the heterogeneous fleet vehicle routing problem. *Computers and Operations Research* 34, 7, 2080–2095.

Christofides, N., Eilon, S., 1969. An algorithm for the vehicle-dispatching problem. *Operational Research Quarterly* 20, 3, 309–318.

Clarke, G., Wright, J., 1964. Scheduling of vehicles from a central depot to a number of delivery points. *Operations Research* 12, 4, 568–581.

Cuervo, D.P., Goos, P., Sörensen, K., Arráiz, E., 2014. An iterated local search algorithm for the vehicle routing problem with backhauls. *European Journal of Operational Research* 237, 2, 454–464.

Deif, I., Bodin, L., 1984. Extension of the Clarke and Wright algorithm for solving the vehicle routing problem with backhauls. Proceedings of the Babson Conference on Software Uses in Transportation and Logistics Management, Babson Park, MA, pp. 75–96.

Dominguez, O., Juan, A.A., Faulin, J., 2014. A biased-randomized algorithm for the two-dimensional vehicle routing problem with and without item rotations. *International Transactions in Operational Research* 21, 3, 375–398.

Faulín, J., Juan, A.A., 2008. The ALGACEA-1 method for the capacitated vehicle routing problem. *International Transactions in Operational Research* 15, 5, 599–621.

Fisher, M., 1994. Optimal solution of vehicle routing problems using minimum K-trees. *Operations Research* 42, 626–642.

Gajpal, Y., Abad, P.L., 2009. Multi-ant colony system (MACS) for a vehicle routing problem with backhauls. *European Journal of Operational Research* 196, 1, 102–117.

Gendreau, M., 1999. A tabu search heuristic for the heterogeneous fleet vehicle routing problem. *Computers & Operations Research* 26, 1153–1173.

Gillett, B., Miller, L., 1974. A heuristic algorithm for the vehicle-dispatch problem. *Operations Research* 22, 2, 340–349.

Goetschalckx, M., Jacobs-Blecha, C., 1989. The vehicle routing problem with backhauls. *European Journal of Operational Research* 42, 39–51.

Goetschalckx, M., Jacobs-Blecha, C., 1993. The vehicle routing problem with backhauls: properties and solution algorithms. Technical Report MHRC-TR-88-13. Georgia Institute.

Golden, B., Alfaro, J., Baker, E., Schaffer, J., 1985. The vehicle routing problem with backhauling: two approaches. Proceedings of the 21st Annual Meeting of the Southeast Institute of Management Science, Myrtle Beach, SC, pp. 90–92.

Golden, B., Assad, A., Levy, L., 1984. The fleet size and mix vehicle routing problem. *Computers & Operations Research* 11, 49–66.

Gonçalves, J.F., Resende, M.G., 2014. An extended Akers graphical method with a biased random-key genetic algorithm for job-shop scheduling. *International Transactions in Operational Research* 21, 2, 215–246.

Gonçalves, J.F., Resende, M.G., Dias C.M., 2016. A biased random-key genetic algorithm for the minimization of open stacks problem. *International Transactions in Operational Research* 23, 25–46.

Hemmati, A., Hvattum, L.M., 2017. Evaluating the importance of randomization in adaptive large neighborhood search. *International Transactions in Operational Research* 24, 5, 929–942.

Hoff, A., Andersson, H., Christiansen, M., Hasle, G., Løkketangen, A., 2010. Industrial aspects and literature survey: fleet composition and routing. *Computers & Operations Research* 37, 12, 2041–2061.

Juan, A., Faulin, J., Caceres, J., Barrios, B., Martinez, E., 2014. A successive approximations method for the heterogeneous vehicle routing problem: analyzing different fleet configurations. *European Journal of Industrial Engineering* 8, 6, 762–788.

Juan, A., Faulin, J., Grasman, S., Rabe, M., Figueira, G., 2015a. A review of simheuristics: extending metaheuristics to deal with stochastic optimization problems. *Operations Research Perspectives* 2, 62–72.

Juan, A., Faulin, J., Jorba, J., Riera, D., Masip, D., Barrios, B., 2011. On the use of Monte Carlo simulation, cache and splitting techniques to improve the Clarke and Wright savings heuristics. *Journal of the Operational Research Society* 62, 6, 1085–1097.

Juan, A., Pascual, I., Guimarans, D., Barrios, B., 2015b. Combining biased randomization with iterated local search for solving the multidepot vehicle routing problem. *International Transactions in Operational Research* 22, 4, 647–667.

Lee, Y.H., Kim, J.I., Kang, K.H., Kim, K.H., 2008. A heuristic for vehicle fleet mix problem using tabu search and set partitioning. *Journal of the Operational Research Society* 59, 6, 833–841.

Lin, S., 1965. Computer solutions of the travelling salesman problem. *Bell System Technical Journal* 44, 2245–2269.

Liu, F.-H., Shen, S.-Y., 1999. The fleet size and mix vehicle routing problem with time windows. *Journal of the Operational Research Society* 50, 7, 721–732.

Morán-Mirabal, L.F., González-Velarde, J.L., Resende, M., 2014. Randomized heuristics for the family traveling salesperson problem. *International Transactions in Operational Research* 21, 41–57.

Osman, I., Wassan, N., 2002. A reactive tabu search for the vehicle routing problem with backhauls. *Journal of Scheduling* 5, 4, 263–285.

Parragh, S.N., Doerner, K.F., Hartl, R.F., 2008. A survey on pickup and delivery problems: Part II: transportation between pickup and delivery locations. *Journal für Betriebswirtschaft* 58, 2, 81–117.

Quintero-Araujo, C.L., Caballero-Villalobos, J.P., Juan, A.A., Montoya-Torres, J.R., 2017. A biased-randomized metaheuristic for the capacitated location routing problem. *International Transactions in Operational Research* 24, 5, 1079–1098.

Reimann, M., Ulrich, H., 2006. Comparing backhauling strategies in vehicle routing using ant colony optimization. *Central European Journal of Operations Research* 14, 2, 105–123.

Reis, R., Ritt, M., Buriol, L.S., Resende, M., 2011. A biased random-key genetic algorithm for OSPF and DEFT routing to minimize network congestion. *International Transactions in Operational Research* 18, 401–423.

Renaud, J., Boctor, F.F., 2002. A sweep-based algorithm for the fleet size and mix vehicle routing problem. *European Journal of Operational Research* 140, 3, 618–628.

Resende, M.G., Ribeiro, C.C., 2010. Greedy randomized adaptive search procedures: advances, hybridizations, and applications. In Gendreau, M. and Potvin, J.Y. (eds) *Handbook of Metaheuristics, Vol. 146.* International Series in Operations Research & Management Science. Springer, New York, pp. 283–319.

Ropke, S., Pisinger, D., 2006. A unified heuristic for a large class of vehicle routing problems with backhauls. *European Journal of Operational Research* 171, 3, 750–775.

Salhi, S., Wassan, N., Hajarat, M., 2013. The fleet size and mix vehicle routing problem with backhauls: formulation and set partitioning-based heuristics. *Transportation Research Part E: Logistics and Transportation Review* 56, 22–35.

Santos, H.G., Toffolo, T.A.M., Silva, C.L.T.F., van den Berghe, G., 2016. Analysis of stochastic local search methods for the unrelated parallel machine scheduling problem. *International Transactions in Operational Research*, doi:10.1111/itor.12316

Sheu, J.-B., Talley, W.K., 2011. Green supply chain management: trends, challenges, and solutions. *Transportation Research Part E: Logistics and Transportation Review* 47, 6, 791–792.

Subramanian, A., Penna, P., 2012. A hybrid algorithm for the heterogeneous fleet vehicle routing problem. *European Journal of Operational Research* 221, 3, 513–520.

Taillard, E.D., 1999. A heuristic column generation method for the heterogeneous fleet VRP. *RAIRO – Recherche Operationnelle* 33, 1, 1–14.

Tarantilis, C.D., Kiranoudis, C.T., Vassiliadis, V.S., 2003. A list based threshold accepting metaheuristic for the heterogeneous fixed fleet vehicle routing problem. *Journal of the Operational Research Society* 54, 1, 65–71.

Toth, P., Vigo, D., 1997. An exact algorithm for the vehicle routing problem with backhauls. *Transportation Science* 31, 4, 372–385.

Toth, P., Vigo, D., 1999. A heuristic algorithm for the symmetric and asymmetric vehicle routing problems with backhauls. *European Journal of Operational Research* 113, 3, 528–543.

Toth, P., Vigo, D., 2014. *Vehicle Routing: Problems, Methods, and Applications* (2nd edn.) Volume 18 of MOS-SIAM Series on Optimization. SIAM, Philadelphia, PA.

Tütüncü, G., 2010. An interactive GRAMPS algorithm for the heterogeneous fixed fleet vehicle routing problem with and without backhauls. *European Journal of Operational Research* 201, 2, 593–600.

Tütüncü, G, Carreto, C., Aker, B., 2009. A visual interactive approach to classical and mixed vehicle routing problems with backhauls. *Omega* 37, 1, 138–154.

Ubeda, S., Arcelus, F.J., Faulin, J., 2011. Green logistics at Eroski: a case study. *International Journal of Production Economics* 131, 1, 44–51.

Wassan, N., 2007. Reactive tabu adaptive memory programming search for the vehicle routing problem with backhauls. *Journal of the Operational Research Society* 58, 12, 1630–1641.

Wassan, N., Salhi, S., Nagy, G., 2009. Solving the mixed VRP with backhauling using ants. In *Power Control and Optimization: Proceedings of the Second Global Conference on Power Control and Optimization*, AIP Publishing, Melville, NY, pp. 248–253.

Zachariadis, E., Kiranoudis, C.T., 2012. An effective local search approach for the vehicle routing problem with backhauls. *Expert Systems with Applications* 39, 3, 3174–3184.